Northwestern University

Mark and Joanna Pinsky Distinguished Lecture Series

The Mathematics of Shuffling Cards

Professor Persi Diaconis

Stanford University

Abstract:

The analysis of the usual method of shuffling cards leads to surprisingly neat results. It also has connections with group theory, combinatorics, and geometry. These expository lectures are aimed at a general mathematical audience and are largely self contained and independent of each other.

I: Basic Shuffling

I will show that it takes seven ordinary riffle shuffles to adequately mix up 52 cards. The mathematics behind this is essentially symmetric function theory and leads to things like Hodge decompositions of Hochschild homology. A variant to type-B shuffles is used to break casino card shuffling machines.

II: Hyperplane Arrangements

The combinatorics of hyperplanes in Euclidean space has a simple random walk interpretation. This generates all kinds of shuffling schemes (Bidigare, Hanlon, Rockmore). Generalizations to walks on the chambers of a building and semigroups (Brown) illuminate calculations originating in library science.

III: Adding Up a List of Numbers

The usual process of carries when adding up a list of numbers can be analyzed using an amazing matrix (Holte). This matrix has a card shuffling interpretation. This leads to new results about addition, shuffling, and the Hilbert series of Veronese subrings. This is joint work with Jason Fulman.

♣ **February 4, 5, 6** ♦ **4:00 PM** ♠ **Lunt Hall, Room 105** ♥