More on NC geometry.

A - an algebra$/k$. NC calculus on "Spec(A)"?

$\Omega^{\bullet, NC} \simeq A \otimes \bar{A}^{\otimes \bullet}$ $\qquad$ $C_\bullet(A) := A \otimes \bar{A}^{\otimes \bullet}$

$d, \iota_\Delta \quad \bullet \geq 0$ $\qquad$ $b, B \quad \bullet \geq 0$ $\qquad$ (A k-flat)

"primary" $\qquad$ ι_Δ, B recovered $\qquad$ "primary"

"naive dR differential" $\qquad$ Homological alg; $A \otimes^{L}_{A \otimes A^{op}} A$

By considering all $f : A \to B$

$$\iota_\Delta(a_0 da_1 \ldots da_n) = \sum_{j=1}^{n} (-1)^{j-1} \left[a_j, da_{j+1} \ldots da_n \cdot a_0 \cdot da_1 \ldots da_{j-1}\right]$$

e.g.

$$\iota_\Delta(a_0 da_1) \overset{?}{=} -[a_0, a_1]$$

$$\iota_\Delta(a_0 da_1 da_2) = [a_1, da_2 \cdot a_0] - [a_2, a_0 da_1]$$

...

$$[d, \iota_f] = id - f_* \quad \forall f : A \to A$$

$$B(a_0\otimes\cdots\otimes a_n) = \sum_{j=0}^{n} (-1)^{nj}\, 1\otimes a_j\otimes\cdots\otimes a_n\otimes a_0\otimes\cdots\otimes a_{j-1}$$

$$B(a_0) = 1\otimes a_0$$

$$B(a_0\otimes a_1) = 1\otimes a_0\otimes a_1 - 1\otimes a_1\otimes a_0$$

$$[b, B_f] = \mathrm{id} - f_* \qquad f_* : A \to \underset{\text{ring}}{B}$$

diff'l

Arrange into a double complex:

$C_\cdot(A)[[u]],\ b+uB$ u- formal variable.

$\|$

$CC^-_\cdot(A)$

$$\begin{tikzcd}
\arrow[d] & \arrow[d] & & \\
C_2 \arrow[r] \arrow[d] & uC_3 \arrow[r] \arrow[d] & u^2C_4 & \\
C_1 \arrow[r] \arrow[d] & uC_2 \arrow[r] \arrow[d] & u^2C_3 \arrow[d] & \\
C_0 \arrow[r] & uC_1 \arrow[r] \arrow[d] & u^2C_2 \arrow[d] & \\
 & uC_0 \arrow[r] & u^2C_1 \arrow[d] & \\
 & & u^2C_0 &
\end{tikzcd}$$

+1

0

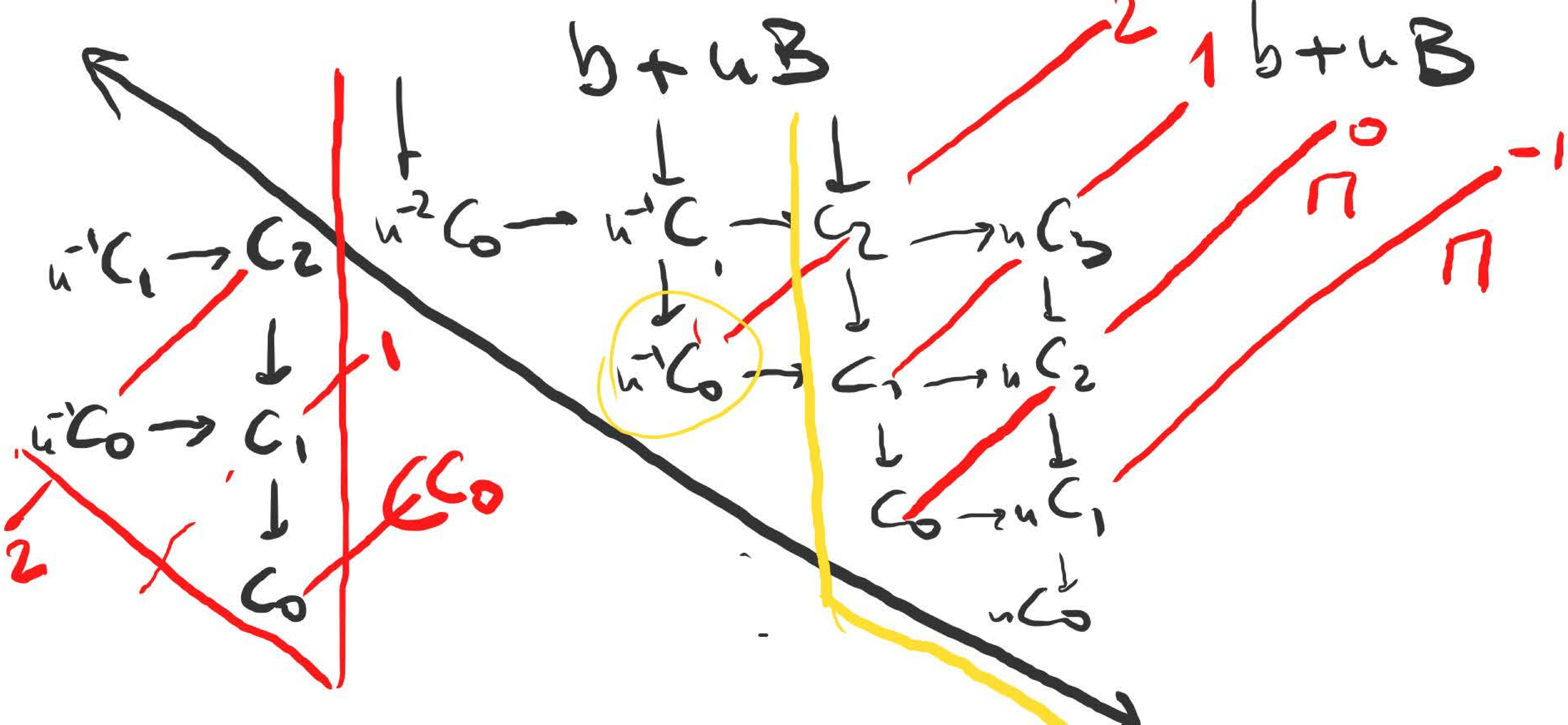
-1

TAKE Π not $\oplus$

$$CC_\bullet^-(A) \longrightarrow CC_\bullet^{per}(A) \to CC_{\bullet-2}$$

$$\| \qquad\qquad\qquad\qquad \|$$

$$C_\bullet(A)[[u]] \longrightarrow C_\bullet(A)((u))$$



$$CC_\bullet(A) = C_\bullet(A)[u^{-1}]$$

$$\parallel$$

$$CC_\bullet(A)((u)) \big/ u^{-1}C_\bullet[u^{-1}]$$

$$
\begin{tikzcd}
u^{-2}C_1 \arrow[r] \arrow[d] & u^{-1}C_2 \arrow[r] \arrow[d] & {} \arrow[d] \\
u^{-2}C_0 \arrow[r] & u^{-1}C_1 \arrow[r] \arrow[d] & C_2 \arrow[d] \\
 & u^{-1}C_0 \arrow[r] & C_1 \arrow[d] \\
 & & C_0
\end{tikzcd}
$$

3 2 1 0

$$CC_\bullet(A) \xrightarrow[\deg 2]{} u^{-1}CC_\bullet^{per} \big/ CC_\bullet^{-}$$

ι_Δ feels like a BV operator if?

$$\begin{tikzcd}
C_\bullet(A)[[u]] \arrow[r, "HKR^{nc}"', "\simeq \text{ if } ((u))"] \arrow[dr, "HKR", "\simeq \text{ reg if } A \text{ Noeth}"'] & \Omega^{\bullet, nc}_{A/k}[[u]] \arrow[d, "proj"] \\
& \Omega^{\bullet}_{A/k}[[u]]
\end{tikzcd}$$

$b + uB$

$\iota_\Delta + ud$

$0 + ud$

$\boxed{\mathbb{Q} \subset k}$

$u \leftrightsquigarrow \hbar$

$u \in H^2_{S^1}(pt)$

(A comm)

$J^n\Omega^\bullet$

$d: J^{n+1}\Omega^\bullet$

in general?

$J \subset P \simeq k[x_1, \dots, x_N]$

$\downarrow$

A

$\boxed{(\hat{\Omega}^\bullet_{P/k}, d)} \xleftarrow[n \to \infty]{} \lim \Omega^\bullet_{P/k} / J^{n+1}\Omega^\bullet$

$J \hookrightarrow P \twoheadrightarrow A$

$\mathbb{Q} \subset k$

$$CC^{per}_\bullet(P)^{\wedge}_J \xrightarrow[\sim]{HKR} \widehat{\Omega}^\bullet_{P/k, J}$$

$$\downarrow$$

$$CC^{per}_\bullet(A)$$

Goodwillie '85?

$\not\exists X$

HKRo

$id - \oint X =$

$= [b + Bu, t]$

the fact that HKR : $\xrightarrow{\sim}$ extends to $\wedge_J$?

filtration : $\oplus J^{k_0} \otimes \cdots \otimes J^{k_n}$ on $C_n(P)$

PERIODIC CYCLIC HOMOLOGY IS RIGID

Towards rigidity of periodic cyclic complexes.

motivation:

$$\begin{tikzcd} CC_\bullet^{per}(P) \arrow[d] \\ CC_\bullet^{per}(P/J) \arrow[u, bend left] \end{tikzcd}$$

say, J nilpotent.

$$\begin{tikzcd} CC_\bullet^{per}(P)^\wedge_J \arrow[d] \\ CC_\bullet^{per}(P/J) \arrow[u, bend left] \end{tikzcd}$$

homotopy inverse ?

Rigidity of periodic cyclic complexes.

Motivation:

$$\begin{tikzcd}
CC^{per}_\bullet(P)^\wedge_J \arrow[loop left] \arrow[d] \\
CC^{per}_\bullet(A) \arrow[loop left] \arrow[u, bend left]
\end{tikzcd}$$

need: homotopy inverse + the homotopies.

So: *what* acts on $CC^{per}_\bullet$? And *what* makes it rigid/topological?

Motivation from calculus:

$\xi \in \mathrm{Vect}(X)$ $\quad L_\xi : \Omega^i \Omega$

$L_\xi = [d, \iota_\xi]$ $\quad \iota_\xi : \Omega^i \to \Omega^{i-1}$

$\Rightarrow$ if $g : X \xrightarrow{\simeq} X$ isotopic to id_X

then $g^* \sim \mathrm{id}$ on H_{dR}

(b/c $\quad e \longrightarrow g \quad \dot{g}_t \cdot g_t^{-1} = \xi_t \; ; \ldots$)

now: $D \in \mathrm{Der}(A)$

$L_D : C_\cdot(A) \to C_\cdot(A)$

$a_0 \otimes \ldots \otimes a_n \longmapsto \sum_{j=0}^{n} a_0 \otimes \ldots \otimes Da_j \otimes \ldots \otimes a_n$

(if D "infinitesimal homomorphism f" then L_D is infin. f_*).

$\iota_D : C_\cdot(A) \to C_{\cdot-1}(A)$

$a_0 \otimes \ldots \otimes a_n \longmapsto a_0 D(a_1) \otimes a_2 \otimes \ldots \otimes a_{n-1}$

$[b, \iota_D] = 0$

Reinhart '63: (main tool of Goodwillie '85)

$$[b + uB, \iota_D + uS_D] = uL_D$$

$S_D(a_0 \otimes \ldots \otimes a_n) =$ "infinitesimal B_f"

$$= \sum_{0 \le j < k \le n} (-1)^{jn} \; a_j \otimes \ldots \otimes D(a_k) \otimes \ldots \otimes a_n \otimes a_0 \otimes \ldots \otimes a_{j-1}$$