More to calculus: Motivation :

$$\Lambda^{\bullet}_{\mathcal{O}_X} T_X \qquad \mathfrak{g}^{\bullet}_X = \Lambda^{\bullet+1}_{\mathcal{O}_X} T_X \text{ — multivector fields}$$

$$\pi \in \Lambda^k \qquad \sum f_{i_1 \dots i_k}(x) \frac{\partial}{\partial x_{i_1}} \wedge \dots \wedge \frac{\partial}{\partial x_{i_k}}$$

$$\iota_\pi : \Omega^{\bullet}_X \to \Omega^{\bullet - k}_X$$

$$L_\pi \doteq [d, \iota_\pi] : \Omega^{\bullet}_X \to \Omega^{\bullet - 1 + k}_X$$

$$[L_\pi, L_\rho] = L_{[\pi, \rho]}$$

$$[L_\pi, \iota_\psi] = \pm \iota_{[\pi, \psi]}$$

$$\dots$$

$$[\,,\,] : \Lambda^{n+1} T_X \otimes \Lambda^{m+1} T_X \to \Lambda^{n+m+1} T_X$$

$$\mathfrak{g}^{\bullet}_{A},\ \delta,\ [\cdot,\cdot]$$

$$\|$$

$$C^{\bullet+1}(A,A)$$

$$\cap$$

$$C^{\bullet+1}_{\text{full}}$$

$$\Rightarrow\ m(a,b)=ab$$

$$\tfrac{1}{2}[m,m]=0$$

$$\delta\varphi+\tfrac{1}{2}[\varphi,\varphi]=0$$

when $\delta=0$:

$$\tfrac{1}{2}[m,m]=0$$

AND BTW:

$$\boxed{\delta=[m,?]}$$

$\Leftrightarrow$ $m+\varphi$ is associative.

Clearly, there is quite a bit.

And, clearly, on the nose: not that great. $[\iota_\varphi, \iota_\psi] \neq 0 \ldots$

$[L_\varphi, \iota_\psi] \neq \iota_{[\varphi,\psi]} \ldots$

hope: yes, up to homotopy.

Would like to extend this to Hoch. chains.

$$L_\varphi(a_0 \otimes a_1 \otimes \dots \otimes a_n) = \underbrace{a_0\varphi(a_1, \dots, a_k)} \otimes a_{k+1} \otimes \dots \otimes a_n$$

$$= \underbrace{\varphi(a_{n-k+1}, \dots, a_n)} \cdot a_0 \otimes a_1 \otimes \dots \otimes a_{n-k}$$

homotopy btw the two.

$$\boxed{\begin{aligned}[m_1, \cdot] &= \delta \\ L_m &= b\end{aligned}}$$

(OR)

Can write L_φ explicitly: $\boxed{b = L_m}$

What acts on $C_\bullet(A)$ or $CC_\bullet^{-}(A)$?

(recall: if want to prove rigidity for periodic cyclic cplexes, need some operations on them).

1) $\mathfrak{g}_A^\bullet, [\,,\,], \delta \qquad (=C^{\bullet+1}(A,A))$ the dgla (L_∞)

$\Rightarrow U(\mathfrak{g}_A^\bullet), \delta$ — dga (need the iotas, too).

$U(\mathfrak{g}_A) \supset \overline{U}(\mathfrak{g}_A)$ the augmentation ideal

Define a dga over $k[\![u]\!]$:

the cobar diffl

Generators: $\cancel{x} \in \overline{U}$ **Differential:**

of deg $|\cancel{x}| \overset{?}{\pm} 1$ $\longrightarrow$ $(\cancel{x}), \cancel{x} \in \overline{U}$

$\mathfrak{g}$

Relations:

b

$(x) \mapsto \sum \pm (x^{(1)})(x^{(2)})$

where $\Delta x = \sum x^{(1)} \otimes x^{(2)}$

$\mathfrak{g}$

$[X, (x)] = (\mathrm{ad}_X(x))$

$\mathcal{B}$

the $u \in \mathcal{U}$ form the dg subalgebra

$u \cdot x$

$x \in U; \quad (x), x \in \overline{U}$

$X \in \mathfrak{g}$

$x \in \overline{U}$

$\big\downarrow \quad [X, (x)] = (\mathrm{ad}_X (x))$

dg subalg

$(x) \longmapsto (\underbrace{\delta x}_{\text{diff'l in } \mathfrak{g}_A}) + \sum \pm (x^{(1)})(x^{(2)}) + + u \cdot X$

$\Lambda^\bullet T$ $\quad \cdots \quad$ $C^\bullet(A, A)$

$\pi \in \Lambda^2 T$ $\quad$ —— $\quad$ $\varphi \in C^2$

$[\pi, \pi] = 0$ $\quad$ $\delta = 0$ $\quad$ $\frac{1}{2}[\varphi, \varphi] = 0$

$\delta\varphi + \frac{1}{2}[\varphi, \varphi] = 0$

$\{f, g\}_\pi = \langle \pi, df \wedge dg \rangle$

$\{f\{g\, h\}_\pi\}_\pi + \circlearrowleft = 0$ $\quad$???

$$\hbar C^\bullet(A,A)[[\hbar]]$$

$$\parallel$$

$$M$$

$$\delta M + \frac{1}{2}[M,M] = 0$$

$\lim_{\hbar \to 0} \frac{1}{\hbar}(a * b - b * a)$
is a Poisson bracket

$$a * b = M(a,b) = ab + \hbar M_1(a,b) + \dots$$

on $A[[\hbar]]$.

associative