General question

$\Delta \subset \Lambda$ (= cyclic category. e.g.)

$E : \Lambda^{op} \to \mathcal{C}$ (sets, Ab grp, Spaces, ...) has colims, ... lims...

we know:
$N\Lambda^{op} \simeq B\mathbb{T}$ $(\mathbb{T} = S^1)$

Can we make out of E a functor $\Lambda \to \Delta^{op}\mathcal{C}$, i.e. $N\Lambda \to \underbrace{\mathcal{N}(\Delta^{op}\mathcal{C})}_{\text{appropriate homotopy nerve}}$

Naive way:
$E: \quad N\Lambda^{op} \to \mathcal{C} \to \Delta^{op}\mathcal{C}$

BUT: if $p : \Lambda^{op} \to *$, then
p_* sends this functor to $R\lim_{\Lambda^{op}} E$
(the cyclic cohomology, if $\mathcal{C} = Ab$).
And we want it to be $HC_\bullet^-$.
So, we should use $p_* : \Lambda^{op}\mathcal{C} \to \Delta^{op}\mathcal{C}$,
but what is the appropriate ∞-categ. interpretation?

For example:

Again, since $E \times_{\Delta} \Lambda$ is "induced", not "free", $\times_{\Lambda}$ seems wrong

$$
\begin{tikzcd}
E \times_{\Delta} \Lambda \times^{h}_{\Lambda} * \arrow[d, "\mathrm{proj}"] & E \times_{\Delta} \Lambda \times_{\Delta} \Lambda \times^{h}_{\Delta} * \arrow[l, shift left] \arrow[l, shift right] & \cdots \arrow[l] \arrow[l, shift left=2] \arrow[l, shift right=2] \\
* \times_{\Delta} \Lambda \times^{h}_{\Lambda} * & * \times_{\Delta} \Lambda \times_{\Delta} \Lambda \times^{h}_{\Lambda} * \arrow[l, shift left] \arrow[l, shift right] & \cdots \arrow[l] \arrow[l, shift left=2] \arrow[l, shift right=2]
\end{tikzcd}
$$

(or some other version of $N\Lambda$, i.e. $B\mathbb{T}$)

is the one on the top an ∞-category?

is the morphism a cocartesian fibration?

And then we know that it defines a functor?

Is there a shorter/more explicit way to see this?