Math 285-1 **Test 1: October 24, 2007** **C. Robinson**

No calculators, no books, no notes
Show all your work in your bluebook. Start each problem on a new page.

1. (18 Points) Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -2 & -2 \\ 2 & 2 & -1 & 0 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ -4 \\ 9 \end{bmatrix}.$$

Find the general parametric vector solution of $\mathbf{A}\mathbf{x} = \mathbf{b}$.

2. (18 Points) Find the inverse of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

3. (14 Points) Give the standard matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that satisfies $T(\mathbf{e}^1) = 2\mathbf{e}^1 + \mathbf{e}^2$ and $T(\mathbf{e}^1 + \mathbf{e}^2) = \mathbf{e}^1 + \mathbf{e}^2$.

4. (20 Points) Complete the sentences below by defining the italicized term. Do not quote a theorem giving conditions equivalent to the definition; give the definition itself.

a. A function $T : \mathbb{R}^n \to \mathbb{R}^m$ is a *linear transformation* provided that ...

b. A set of vectors $\{\mathbf{v}^1, \dots, \mathbf{v}^p\}$ in $\mathbb{R}^m$ is *linearly dependent* provided that ...

5. (30 Points) Indicate which of the following statements are always true (T) and which are false (F). Justify each answer by a counterexample or explanation. Refer to any theorem by an informal statement, not by a theorem numbers.

a. If $\mathbf{A}$ is an $m \times n$ matrix and the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$ is consistent for some $\mathbf{b} \in \mathbb{R}^m$, then the columns of $\mathbf{A}$ span $\mathbb{R}^m$.

b. If $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ in some $\mathbb{R}^n$, then $\operatorname{span}\{\mathbf{u}, \mathbf{v}\} = \operatorname{span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$.

c. If the system $\mathbf{A}\mathbf{x} = \mathbf{b}$ has a unique solution, then $\mathbf{A}$ must be a square matrix.

d. Any set of three vectors $\{\mathbf{v}^1, \mathbf{v}^2, \mathbf{v}^3\}$ in $\mathbb{R}^2$ are linearly dependent.

e. If $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are matrices for which $\mathbf{A}\mathbf{B} = \mathbf{C}$ and $\mathbf{C}$ has 2 columns, then $\mathbf{A}$ has 2 columns.

f. If $\mathbf{A}$ is a 5×3 matrix and $\mathbf{C}$ is a 3×5 matrix such that $\mathbf{C}\mathbf{A} = \mathbf{I}$, then the linear transformation $\mathbf{x} \mapsto \mathbf{A}\mathbf{x}$ is one-to-one.