HOUR TEST 1

- 1. Complete the sentences below by defining the italicized term. Do not quote a theorem giving conditions equivalent to the definition; give the definition itself.
 - (a) A linear transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if
 - (b) The set of vectors $\{\mathbf{v}_1, \dots, \mathbf{v}_p\} \subset \mathbb{R}^n$ is linearly independent if
- 2. Give an example of each of the following:
 - (a) A pair of matrices A, B, neither of which is the **0**-matrix, for which $AB = \mathbf{0}$.
 - (b) A 3×4 matrix which is in echelon form, but not in row reduced echelon form.
 - (c) A set of vectors which spans \mathbb{R}^4 and is also linearly independent.
- 3. Find the standard matrix of each the following transformations.
 - (a) The vertical shear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, that maps \mathbf{e}_1 into $\mathbf{e}_1 2\mathbf{e}_2$ and leaves \mathbf{e}_2 unchanged.
 - (b) The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, that reflects points through the (horizontal) x_1 -axis.
 - (c) The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, that reflects points through the line $x_1 = x_2$.
 - (d) The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, that first reflects points through the (horizontal) x_1 -axis and then reflects them through the line $x_1 = x_2$.
- 4. Indicate whether each of the following statements is true or false (in general) by placing a check mark in the correct box to the left of the statement. If the statement is false, give an example showing that it is false. If the statement is true, explain why it is true. You may quote theorems from the text.

True False

- (a) If A is an $m \times n$ matrix and B is an $n \times p$ matrix, each column of the matrix AB is a linear combination of the columns of A.
 - True False
- (b) If A is a 14 × 17 matrix, then the linear transformation $T_A : \mathbb{R}^{17} \to \mathbb{R}^{14}$ (defined by $T_A(\mathbf{x}) = A\mathbf{x}$) cannot be one-to-one.

True False

(c) If B is a square $m \times m$ matrix and the equation $B\mathbf{x} = \mathbf{0}$ has only the trivial solution, then $B\mathbf{x} = \mathbf{b}$ is consistent for any $\mathbf{b} \in \mathbb{R}^m$.

(d) True False
(d) True False
The set of vectors
$$S = \left\{ \begin{bmatrix} 1\\2\\3\\4 \end{bmatrix}, \begin{bmatrix} 6\\7\\8\\9 \end{bmatrix}, \begin{bmatrix} 0\\0\\0\\0 \end{bmatrix}, \begin{bmatrix} 2\\3\\5\\7 \end{bmatrix} \right\}$$
 spans \mathbb{R}^4 .

5. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation. Use the definition of linear transformation to prove that $T(\mathbf{0}) = \mathbf{0}$ (where the first $\mathbf{0} \in \mathbb{R}^n$ and the second $\mathbf{0} \in \mathbb{R}^m$).