Math 285-1 **Test 2: November 14, 2006** **C. Robinson**

No calculators, no books, no notes — You do not need to simplify your answer

Show all your work in your bluebook. Start each problem on a new page.

1. (12 Points) Calculate the determinant $\det \begin{bmatrix} 1 & -2 & 1 & 4 \\ 2 & -4 & 4 & 8 \\ 3 & -4 & 2 & 5 \\ 0 & 2 & -4 & -9 \end{bmatrix}$.

2. (24 Points) The matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & -1 & 0 & 0 & 2 \\ 2 & 1 & 6 & 0 & 1 \end{bmatrix}$ has the reduced echelon form $\mathbf{U} = \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

a. Find a basis for the nullspace of A.

b. Find a basis for the column space of A.

c. Find a basis for the row space of A.

3. (32 Points) Indicate which of the following statements are always true and which are false (not always true). If the statement is true, give a SHORT justification. If the statement is false, give a SHORT counterexample or explanation. Use complete sentences. Refer to any theorem by an informal statement, not by a theorem number.

a. If $\mathbf{A}$ and $\mathbf{B}$ are $n \times n$ matrices with $\det(\mathbf{B}) \neq 0$, then $\det(\mathbf{A}\mathbf{B}^{-1}) = \det(\mathbf{A})/\det(\mathbf{B})$.

b. If the columns of a 3×3 matrix $\mathbf{A}$ determine a parallelepiped in $\mathbb{R}^3$ of volume 10, then the volume of the parallelepiped determined by the columns of $2\mathbf{A}$ is 20.

c. If $\mathbf{A}$ is a 3×5 matrix with rank 2, then the dimension of the null space of $\mathbf{A}$ is 2.

d. If $\mathbf{A}$ is a 5×3 matrix, then the rows of $\mathbf{A}$ must be linearly dependent.

e. Let $\mathbf{W}$ be a subspace of $\mathbf{V}$, $\dim(\mathbf{W}) = 4$, and $\dim(\mathbf{V}) = 7$. Every basis of $\mathbf{W}$ can be extended to a basis of $\mathbf{V}$ by adding three more vectors to it.

f. Let $\mathbf{W}$ be a subspace $\mathbf{V}$, $\dim(\mathbf{W}) = 4$, and $\dim(\mathbf{V}) = 7$. Every basis $\mathscr{B}$ of $\mathbf{V}$ can be reduced to a basis of $\mathbf{W}$ by removing three vectors from $\mathscr{B}$.

g. The set of all polynomials $p(t)$ with the property that $p(1) = 1$ is a subspace of the vector space $\mathbb{P}$ of all polynomials.

h. If $\mathbf{H}$ is a subspace of $\mathbb{R}^3$ that is not the zero subspace, $\mathbf{H} \neq \{\mathbf{0}\}$, then there is a matrix $\mathbf{A}$ such that $\mathbf{H} = \mathrm{Col}(\mathbf{A})$.

4. (14 Points) Assume $\mathbf{A}$ is an $m \times n$ matrix and $\mathbf{V} \subset \mathbb{R}^n$ is a subspace. Show directly from the definition of a subspace that the set $\mathbf{A}(\mathbf{V}) = \{\, \mathbf{A}\mathbf{v} : \mathbf{v} \in \mathbf{V} \,\}$ is a subspace of $\mathbb{R}^m$.

5. (18 Points) Consider the set of polynomials $\mathbf{S} = \{1+t^2, 1+2t, 1+3t^2\}$ in $\mathbf{P}_2$, the set of polynomials of degree ≤ 2.

a. Prove that the set $\mathbf{S}$ is linearly independent.

b. Prove that $\mathbf{S}$ is a basis for $\mathbb{P}_2$.

c. Find the coordinate vector $[p(t)]_{\mathbf{S}}$ of the polynomial $p(t) = 7 + 4t + 9t^2$ relative to the basis $\mathbf{S}$.