Math 285-1 **Test 2: November 19, 2007** **C. Robinson**

No calculators, no books, no notes
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1. (16 Points) *Calculate* the determinant $\det \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 2 & 4 & 6 \\ 1 & 1 & 2 & -1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$.

2. (24 Points)

The matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 & -1 & 3 & 9 \\ -1 & 0 & -2 & 0 & 1 \\ 1 & 0 & 2 & 1 & 2 \\ 2 & 1 & 1 & 1 & 2 \end{bmatrix}$ has the reduced echelon form $\mathbf{U} = \begin{bmatrix} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & -3 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

a. *Give* a basis for the nullspace of **A** and its dimension.

b. *Give* a basis for the column space of **A** and its dimension.

c. *Give* a basis for the row space of **A** and its dimension.

3. (18 Points) Assume that $\mathbf{v}^1$, $\mathbf{v}^2$, and $\mathbf{v}^3$ are three nonzero vectors in $\mathbb{R}^n$ such that $5\mathbf{v}^1 + 3\mathbf{v}^2 - \mathbf{v}^3 = \mathbf{0}$ and such that no pair of vectors is parallel. *Find* a basis of $\mathbf{W} = \mathrm{Span}\{\mathbf{v}^1, \mathbf{v}^2, \mathbf{v}^3\}$ and *explain* why it is a basis.

4. (18 Points) Assume that $T : \mathbf{V} \to \mathbf{W}$ is a one-to-one linear transformation between the vector spaces **V** and **W** and that $\{\, \mathbf{v}^1, \dots, \mathbf{v}^k \,\}$ is a set of linearly independent vectors in **V**. *Prove* that $\{\, T(\mathbf{v}^1), \dots, T(\mathbf{v}^k) \,\}$ is a set of linearly independent vectors in **W**.

5. (24 Points) *Indicate* which of the following statements are always true (T) and which are false (F). *Justify* each answer by a counterexample or explanation. Refer to any theorem by an informal statement, not by a theorem numbers.

a. If $\mathbf{v}^1$ and $\mathbf{v}^2$ are vectors in $\mathbb{R}^2$ which determine a parallelogram of area 3 and **A** is a 2×2 matrix with determinant 5, then $\mathbf{A}\mathbf{v}^1$ and $\mathbf{A}\mathbf{v}^2$ determine a parallelogram of area 8.

b. If **A** is an 3×3 matrix with $\mathbf{A}^3 = \mathbf{0}$, then $\det(\mathbf{A}) = 0$.

c. If **A** is an 3×3 matrix, then $\det(-\mathbf{A}) = -\det(\mathbf{A})$.

d. Some subset of the rows of a matrix **A** form a basis of the row space of **A**.

e. There is a basis of $\mathbb{P}_5$, the polynomials of degree less than or equal to five, that includes the two polynomials $p_1(t) = 1 + t^2 + t^4$ and $p_2(t) = t + t^3$.

f. If **A** is an $m \times n$ matrix with $\mathrm{rank}(\mathbf{A}) = m$, then the transformation $\mathbf{x} \mapsto \mathbf{A}\mathbf{x}$ is one-to-one.