# Math 313-1 Test 1 October 21, 2003 R.C. Robinson

No books or notes allowed. Calculators are allowed.
Show all your work in your bluebook. Start each problem on a new page.

**1**. (25 Points) Let

$$f(x) = \frac{1}{3}x^3 - \frac{1}{3}x^2 + \frac{1}{3}\,x.$$

**a**. Find the fixed points and classify each of them as attracting, repelling, or neither.
**b**. Use graphical method of iteration to determine the basin of attraction of all the attracting fixed points. (There are no critical points.)

**2**. (25 Points) Let

$$f(x) = 1 - \frac{10}{9}x^2.$$

**a**. Determine whether the period-2 orbit

$$\left\{ \frac{9}{20} \pm \frac{3\sqrt{13}}{20} \right\}$$

is attracting or repelling.
**b**. Show that $y = C(x) = \frac{1}{3}x + \frac{1}{2}$ is a conjugacy from $f(x)$ to $g(y) = \frac{10}{3}y(1-y)$.

**3**. (25 Points) Consider $f_r(x) = r\sin(x)$ with $r > 0$.
**a**. Show that $f$ has negative Schwarzian derivative.
**b**. For $1 < r < \pi$, the interval $[0, \pi]$ is positively invariant by $f_r$, i.e., $f_r([0,\pi]) \subset [0,\pi]$. For $1 < r < \pi$, explain, why any attracting periodic orbit must contain the contain $x_c = \frac{\pi}{2}$ in its basin. How many attracting periodic orbits can there be?

**4**. (25 Points) Consider the one parameter family of maps given by

$$f_\mu(x) = \mu\, x^2 - 1.$$

**a**. What are the fixed points? For which $\mu$ do real fixed points exist?
**b**. What parameter value $\mu$ is a potential tangential bifurcation value and what is the fixed point? You do not need to check all the parameters, but check the conditions on $f_\mu'(x)$ at the fixed point.
**c**. What parameter value $\mu$ is a potential period doubling bifurcation value and what is the fixed point? You do not need to check all the parameters, but check the conditions on $f_\mu'(x)$ at the fixed point.