Math 313-1 Test 1, January 28, 2003 R.C. Robinson

No books or notes allowed. Calculators are allowed.

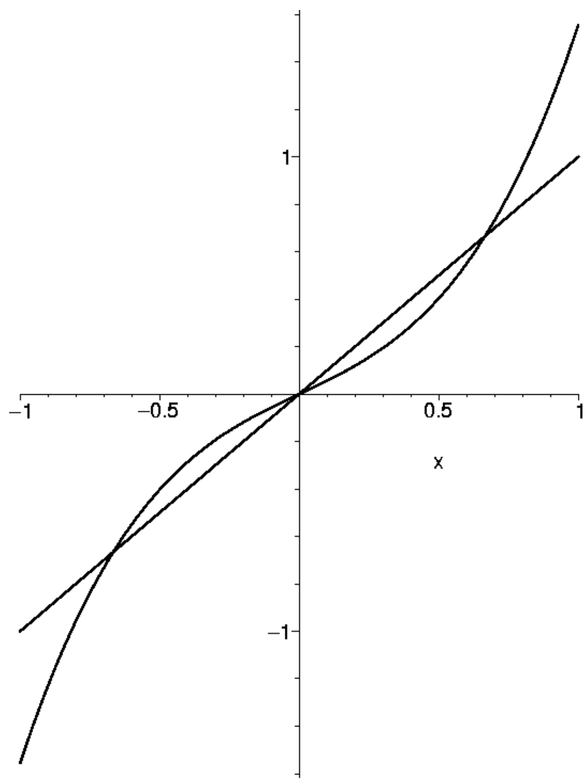
1. Consider the function

$$f(x) = \begin{cases} 3x & \text{for } 0 \le x \le \frac{1}{3} \\ -3x + 2 & \text{for } \frac{1}{3} \le x \le \frac{2}{3} \\ 3x - 2 & \text{for } \frac{2}{3} \le x \le 1. \end{cases}$$

- **a**. (20 Points) This function has 3^n points which are fixed by f^n, $\#\text{Fix}(f^n) = 3^n$. Make a table for $1 \le n \le 6$ showing the following: (i) n, (ii) number of fixed points of f^n, $\#\text{Fix}(f^n)$, (iii) how many of these points fixed by f^n have a lower period, (iv) number of points of period n, $\#\text{Per}(n)$, and (v) number of orbits of period n.
- **b**. (10 Points) Determine whether $\dfrac{1}{6}$ and $\dfrac{1}{4}$ are periodic or eventually periodic.

2. (30 Points) Let

$$f(x) = x^3 + \frac{5}{9}\,x.$$



- **a**. Find the fixed points and classify each of them as attracting, repelling, or neither.
- **b**. Use graphical method of iteration to determine the basin of attraction of all the attracting fixed points.

(over)

3. (15 Points) Let

$$f(x) = x^3 - \frac{5}{4}\,x.$$

The fixed points are 0 and $\pm\frac{3}{2}$. Determine the stability of the period-2 orbit $\left\{\frac{1}{2}, -\frac{1}{2}\right\}$. Note: The period-$2$ points satisfy $f(x) = -x$.

4. (25 Points) Let

$$f(x) = x^3 - \frac{9}{16}\,x.$$

a. Show the Schwarzian derivative of f is negative. Note:

$$S_f(x) = \frac{f'''(x)\,f'(x) - \frac{3}{2}f''(x)^2}{f'(x)^2}.$$

b. Find the critical points.

c. The fixed points are 0 and $\pm\frac{5}{4}$, where 0 is attracting and $\pm\frac{5}{4}$ are repelling. The basin of attraction of 0 is contained between the other two fixed points,

$$\mathcal{B}(0) \subset \left(-\frac{5}{4}, \frac{5}{4}\right).$$

(These are facts you may use without proving.) **Explain** why the critical points must be in the basin of attraction of 0.