Math 313-1 Test 2 November 18, 2003 R.C. Robinson

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1. (25 Points) Assume f is a continuous map on the real line which has a period-5 orbit with $f(1) = 3$, $f(2) = 5$, $f(3) = 4$, $f(4) = 2$, and $f(5) = 1$. Label the interval between these points with symbols 1 through 4 by $\mathbf{I}_j = [j, j+1]$ so $\mathbf{I}_1 = [1, 2]$ etc.
  a. Give the transition graph for these intervals.
  b. What periods are forced to exist by this transition graph?

2. (25 Points) Let

$$T(x) = T_{10}(x) = \begin{cases} 10\,x & x \le 0.5 \\ 10\,(1-x) & x \ge 0.5 \end{cases}$$

be the tent map with slope 10.
  a. Sketch the graph of T.
  b. Consider the set

$$\mathbf{K}_3 = \{x : T^j(x) \in [0,1] \text{ for } 0 \le j \le 3\} = \bigcap_{j=0}^{3} T^{-j}([0,1]).$$

  How many intervals does $\mathbf{K}_3$ contain and what is the length of each of these intervals?
  c. Let

$$\mathbf{K} = \bigcap_{j=0}^{\infty} T^{-j}([0,1]).$$

  Explain the set $\mathbf{K}$ is exactly the set of numbers in $[0,1]$ which have a decimal expansion using only 0's and 9's; that is, a x in $\mathbf{K}$ can be represented as a sum $x = \sum_{j=1}^{\infty} a_j/10^j$ where all the a_j are either 0 or 9.
  d. Give a number in $\mathbf{K}$ which is not an endpoint.

3. (25 Points) Discuss two methods we have used to prove that a map of the interval $[0,1]$ to itself has sensitive dependence on initial conditions. You need only outline the approaches; you do not need to give all the details of the proofs.

4. (25 Points) Consider the map defined by

$$f(x) = \begin{cases} \frac{1}{2} - 2\,x & \text{if } 0 \le x \le \frac{1}{4} \\ 4\,x - 1 & \text{if } \frac{1}{4} \le x \le \frac{1}{2} \\ -\frac{3}{2}x + \frac{7}{4} & \text{if } \frac{1}{2} \le x \le 1. \end{cases}$$

  a. Draw the graph of f? (Not the transition graph, but the graph of $f(x)$ versus x.)
  b. Explain why the intervals $\mathbf{I}_L = [0, 1/4]$, $\mathbf{I}_C = [1/4, 1/2]$, and $\mathbf{I}_R = [1/2, 1]$ form is a Markov partition for f.
  c. What is the transition graph for this Markov partition?
  d. Using the transition graph, determine what periods the function has.
  e. Explain why this map satisfies the conditions of one of the theorem presented to imply that it is topologically transitive on $[0,1]$.