Math 313-1 Test 2 February 25, 2003 R.Clark Robinson

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1. (25 Points) Assume f is a continuous map on the real line which has a period-7 orbit with $f(1) = 3$, $f(2) = 7$, $f(3) = 5$, $f(4) = 6$, $f(5) = 4$, $f(6) = 2$, and $f(7) = 1$. (Note that this is not a Stefan cycle.) Label the interval between these intervals with symbols 1 through 6 by $\mathbf{I}_j = [j, j+1]$ so $\mathbf{I}_1 = [1, 2]$ etc.

a. Give the transition graph for these intervals.
b. What periods are forced to exist by this transition graph?

2. (20 Points) Explain why the map $Q(x) = 4x \pmod 1$ has sensitive dependence on initial conditions.

3. (30 Points) Let

$$T(x) = T_4(x) = \begin{cases} 4\,x & x \le 0.5 \\ 4\,(1-x) & x \ge 0.5 \end{cases}$$

be the tent map with slope 4.

a. Sketch the graph of T.
b. Consider the set

$$\mathbf{K}_3 = \{x : T^j(x) \in [0,1] \text{ for } 0 \le j \le 3\} = \bigcap_{j=0}^{3} T^{-j}([0,1]).$$

How many intervals does $\mathbf{K}_3$ contain and what is the length of each of these intervals?

c. Let

$$\mathbf{K} = \bigcap_{j=0}^{\infty} T^{-j}([0,1]).$$

Explain the set $\mathbf{K}$ is exactly the set of numbers in $[0,1]$ which have a quartic expansion using only 0s and 3s, i.e., the x which can be represented as a sum $x = \sum_{j=1}^{\infty} \frac{a_j}{4^j}$ where all the a_j are either 0 or 3.

d. Give a number in $\mathbf{K}$ which is not an endpoint.

4. (25 Points) Consider the map defined by

$$f(x) = \begin{cases} 1 - 2\,x & \text{if } 0 \le x \le \frac{1}{3} \\ 2\,x - \frac{1}{3} & \text{if } \frac{1}{3} \le x \le \frac{2}{3} \\ 3\,(1-x) & \text{if } \frac{2}{3} \le x \le 1. \end{cases}$$

a. Draw the graph of f? (Not the transition graph, but the graph of $f(x)$ versus x.)
b. What is a Markov partition for f? What is the expanding factor for f on $[0,1]$?
c. What is the transition graph for this Markov partition?
d. Explain why this map satisfies the conditions of one of the theorem presented to imply that it is topologically transitive on $[0,1]$.