Math 303 **Final: December, 2000** **C. Robinson**

Start each problem on a new page. Show your work. Grades will be posted on the webpage under the last numbers of your social security number.

1. (50 Points) Consider the system of differential equations
$$\dot{x} = -x - y + 4$$
$$\dot{y} = 3 - xy$$
which has fixed points at $(1,3)$ and $(3,1)$.
(a) Determine the linearized stability of each fixed point.
(b) Solve for the general solution of the linearized equations at each fixed point.
(c) Draw the phase portrait for the system using the information from parts (a) and (b) and the nullclines. (In particular, which are the direction which contract most strongly?) Explain your sketch of the phase portrait.

2. (30 Points) Consider the system of differential equations
$$\dot{x} = y$$
$$\dot{y} = -4x + y(1 - x^2 - y^2).$$
Show that the system has a periodic orbit.

3. (40 Points) Consider the system of differential equations
$$\dot{x} = y$$
$$\dot{y} = -x - 2\,x^3 - 3\,x^5 + y(2\,\mu + x^2 + y^2).$$
Show there is a Hopf bifurcation for $\mu = 0$.
(a) What are the eigenvalue at the fixed point at the origin for different values of μ?
(b) For $\mu = 0$, is the origin weakly attracting or repelling?
(c) Is it a subcritical or supercritical bifurcation, and is the periodic orbit attracting or repelling? Explain your answer.

4. (50 Points) Consider the system of differential equations given in polar coordinates by
$$\dot{r} = r(1 - r^2)(r^2 - 4)$$
$$\dot{\theta} = 1.$$
(a) Draw the phase portrait in the (x, y)-plane.
(b) What are the attractors for this system? Explain your answer including what properties this sets have to make them attractors.

5. (30 Points) Explain how we know that the Lorenz system has a chaotic attractor for $\sigma = 10$, $b = 8/3$, and $r = 28$. What conditions must be checked to verify this and what methods can be used for this verification? (You do not need to actually verify these conditions.)