(1) (30 Points) Let $f(x) = \frac{1}{2}x^3 - \frac{3}{2}x^2 + 2x$.
 (a) Find the fixed points and classify them as attracting, repelling, or neither.
 (b) Use the cobweb plot analysis to determine the dynamical behavior of all real initial conditions. Describe the orbits using words as well as by the plot.

(2) (30 Points) Let $f(x, y) = (1 - 4x^2 + 4y, x)$. Find the fixed points and classify them as sink, source, saddle, or other type.

(3) (25 Points) Consider the linear map

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \frac{3}{4} & \frac{5}{4} \\ \frac{1}{4} & \frac{7}{4} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

The eigenvalues of the matrix are 2 and 1/2. Describe the dynamics of this linear map. Identify the stable and unstable manifolds.

(4) (30 Points) Let

$$T(x) = \begin{cases} 4x & x \le 0.5 \\ 4(1-x) & x \ge 0.5. \end{cases}$$

 (a) Sketch the graph of T.
 (b) Describe the set of points x such that $x, T(x), T^2(x) \in [0, 1]$,

$$\{x \,:\, T^j(x) \in [0,1] \text{ for } 0 \le j \le 2\}.$$

 It is made up of how many intervals of what length? What is its total length?
 (c) How many intervals make up $\{x \,:\, T^j(x) \in [0,1] \text{ for } 0 \le j \le n\}$; what is the length of each interval; what is the total length of the set?
 (d) What is the total "length" of the set $\{x \,:\, T^j(x) \in [0,1]\ 0 \le j < \infty\}$? Alternatively, what is the total length of the intervals outside the set, i.e., in the intervals

$$[0,1] \setminus \{x \,:\, T^j(x) \in [0,1] \text{ for } 0 \le j < \infty\}?$$

(5) (30 Points) Let f be a continuous function defined on the interval $[1, 9]$ such that $f(1) = 5$, $f(2) = 9$, $f(3) = 8$, $f(4) = 7$, $f(5) = 6$, $f(6) = 4$, $f(7) = 3$, $f(8) = 2$, and $f(9) = 1$, Assume the the function is linear between these integers.
 (a) Label the intervals between the integers and give the transition graph.
 (b) For which n is there a period-n orbit? Give the Symbol sequence in terms of the intervals which will give each period that exists.

(6) (30 Points) Let $f(x) = rx(1 - x)$ for $r = 0.5 + \sqrt{17}/2 \approx 3.06$. This map has a period-2 orbit which is approximately $x_1 = 0.581$ and $x_2 = 0.745$.
 (a) Determine the stability of the period-2 orbit using the values above.
 (b) What is the Lyapunov exponent of x_1?
 (c) Let $x_0 = f(0.5) \approx 0.765$. What is the Lyapunov exponent of x_0? Notice that 0.5 is the critical point.

(7) (25 Points) Give an example of a function with a chaotic orbit. Explain why the orbit is chaotic.