Math 303 Test 1: October 23, 2001 C. Robinson

No books, no notes. Calculators allowed, but you must show your work.

Start each problem on a new page of your blue book.

1. (40 Points) Consider each of the following systems of linear differential equations with the indicated eigenvalues and eigenvectors. For equation example, (i) give the general real solution, and (ii) draw the phase portrait.

   (a)
   $$\dot{\mathbf{x}} = \begin{pmatrix} 3 & 1 \\ 4 & 0 \end{pmatrix} \mathbf{x},$$
   with eigenvalues 4 and -1 with respective eigenvectors
   $$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

   (b)
   $$\dot{\mathbf{x}} = \begin{pmatrix} -5 & -5 \\ 2 & 1 \end{pmatrix} \mathbf{x}.$$
   with eigenvalues $-2 \pm i$, and eigenvector
   $$\begin{pmatrix} 5 \\ -3 \end{pmatrix} + i \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$
   for the eigenvalue $-2 + i$.

2. (20 Points) Consider the equations
   $$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} y \\ 4\,x \end{pmatrix}.$$
   Using the Improved Euler Method, $h = 0.1$, and initial condition $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, find the approximate solution at time 0.2.

3. (20 Points) Consider the differential equation
   $$\dot{x} = (1 - x^2)(x^2 - 4) = -x^4 + 5\,x^2 - 4.$$
   (a) Find the stability type of each fixed point.
   (b) Draw the phase portrait on the real line. Also sketch the graph of $x(t)$ in the (t, x)-space for several representative initial conditions. Describe in words which initial conditions converge to which fixed points.

4. (20 Points) Consider the equations
   $$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} x(2 - x - y) \\ y(1 - x - y) \end{pmatrix}.$$
   (a) Find the three fixed points (including the origin). Classify each of them as stable node, stable focus, saddle, unstable node, etc.
   (b) Draw the phase portraits using the nullclines and the answer to part (a).
   (c) Do most solutions with $x_0 > 0$ and $y_0 > 0$ tend to a fixed point? Which fixed point?