Math 313-1 Test 2 Feb 20, 2002.

(3) $f(x) = x^3$ $\quad g(y) = \frac{1}{4}y^3 + \frac{3}{2}y^2 + 3y$

$y = C(x) = 2x - 2.$

$C \circ f(x) = C(x^3) = 2x^3 - 2$

$g \circ C(x) = g(2x-2) = \frac{1}{4}(2x-2)^3 + \frac{3}{2}(2x-2)^2 + 3(2x-2)$

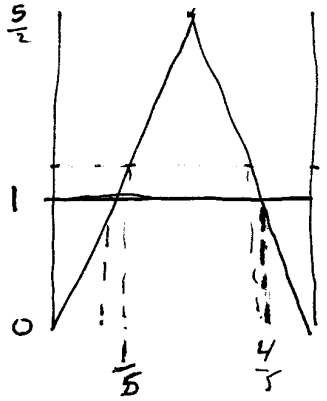
$= 2(x^3 - 3x^2 + 3x - 1) + 6(x^2 - 2x + 1) + 6x - 6$

$= 2x^3 + x^2(-6+6) + x(6 - 12 + 6) + (-2 + 6 - 6)$

$= 2x^3 - 2.$

These are equal, so a conjugacy.

(4)



(b) $[0, \frac{1}{25}] \cup [\frac{4}{25}, \frac{5}{25}] \cup [\frac{20}{25}, \frac{21}{25}] \cup [\frac{24}{25}, 1]$

There are $2^2 = 4$ intervals, each of length $(\frac{1}{5})^2 = \frac{1}{25}$.
Total length $(\frac{2}{5})^2 = \frac{4}{25}$.

(c) All the point in K have expansions base 5 which use 0s and 4s. End pairs end in repeated 0s or repeated 4s.

(d) Alternating 0 & 4 we get

$\frac{0}{5} + \frac{4}{25} + \frac{0}{5^3} + \frac{4}{5^4} + \cdots$

$= \frac{4}{25}\left[1 + \frac{1}{25} + (\frac{1}{25})^2 + \cdots\right]$

$= \frac{4}{25}\left(\frac{1}{1 - \frac{1}{25}}\right) = \frac{4}{25} \cdot \frac{25}{24} = \frac{1}{6}.$

Another pair is

$\frac{4}{5} + \frac{4}{5^3} + \cdots = \frac{4}{5} \cdot \frac{25}{24} = \frac{5}{6}.$