ERRATA AND ADDITIONS FOR THE SECOND EDITION OF DYNAMICAL SYSTEMS: STABILITY, SYMBOLIC DYNAMICS, AND CHAOS

by Clark Robinson

(Preface page 4 L. -13) Yorke (1990) should be Nusse and Yorke (1990).

p. 4 (L. -2) Subsections 8.3.1 – 4

p. 10 (L. -3) "could be combined with the section in Chapter VIII)."

p. 17 (L. 2) $\{(x, f(x)\}$ should read $\{(x, f(x))\}$.

p. 24 (L. -12) should be "compact nested nonempty sets."

p. 24 (L. -4) should be " ... is closed and positively invariant ... "

p. 27 (L. 3) "A non-empty set S ... "

p. 30 (L. -17) should be "$\pi(\sum_{n\geq 1} j_n\, 3^{-n}) = \sum_{n\geq 1} (j_n/2) 2^{-n}$."

p. 33 (Section 2.4.3) Explanation: S. Zeller and M. Thaler ("Almost sure excape from the unit interval under the logistic map", *Amer. Math. Monthly* **108** (2001), pages 155 – 158.) have a simpler proof based on the earlier thesis of S. Zeller ("Chaosbegriffe der topologishen Dynamik", Dipolmarbeit, Salzburg, 1991). The map

$$y = \phi(x) = \frac{2}{\pi} \arcsin \sqrt{x}$$

is a conjugacy between $F_4(x)$ and $g_4(y) = 1 - |1 - 2y|$, $g_4(y) = \phi \circ F_4 \circ \phi^{-1}(y)$. For $\mu > 4$, $F_\mu([0, 1]) = [0, \frac{\mu}{4}]$, so it is natural to scale ϕ by the factor $\frac{\mu}{4}$ to investigate F_μ. Let $\phi_\mu(x) = \frac{\mu}{4}\, \phi(\frac{4}{\mu} x)$. Define the map g_μ by

$$g_\mu(y) = \phi_\mu \circ F_\mu \circ \phi_\mu^{-1}(y).$$

Then a simple calculation shows that $|g'_\mu(y)| \geq \sqrt{\mu}$ for all $y \in [0, \phi_\mu(a)]$, so F_μ has can invariant Cantor set. The proof is a follows. Let $y = \phi_\mu(x)$ for $x \in [0, 1)$. Because ϕ is a conjugacy of F_4 and g_4,

$$\phi'(F_4(x))\, F'_4(x) = (2\, \mathrm{sign}(1 - 2x))\, \phi'(x).$$

Also, $F'_\mu(x) = \frac{a}{4} F'_4(x)$. Thus

$$\begin{aligned} g'_4(y) &= \frac{\phi'_\mu(F_4(x))\, F'_\mu(x)}{\phi'_\mu(x)} = \frac{a\, \phi'(F_4(x))\, F'_4(x)}{4\, \phi'_\mu(x)} \\ &= \mathrm{sign}(1 - 2x))\, \frac{\mu\, \phi'(x)}{2\, \phi'_\mu(x)} \\ &= \mathrm{sign}(1 - 2x))\, \sqrt{\mu}\, \Big(\frac{1 - \frac{4}{\mu}}{1 - x}\Big)^{\frac{1}{2}}. \end{aligned}$$

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

Since $\frac{4}{\mu} < 1$, the last term is greater than $\sqrt{\mu}$.

p. 36 (L. -12) should be "$1 > |(T_2 \circ f \circ T_1)'(0)| = |T_2'(w_0)| \cdot |f'(z_0)| \cdot |T_1'(0)|$"

p. 38: (Section 2.5) Explanation: We are attempting to understand the orbits of all points in the invariant set Λ. At least in a theoretical way, we can determine the periodic points. We also want to show that there are points whose orbit is dense in the cantor set Λ and points with other complicated dynamics. By introducing symbols to describe the location of a point, the dynamics of a point in the Cantor set can be determined by means of a sequence of these symbols. Because many different patterns of symbols can be written down, points with many different types of dynamics can be shown to exist.

p. 43 (Caption on Figure 6.1) S^1 should be S and $[0, 1]$ should be $[0, 2]$.

p. 45: (L. -5)

$$= \lim_{\substack{x \to 1 \\ x > 1}} h_0'(x).$$

p. 50: (L. -23) Explanation: The covering space $\mathbb{R}$ of S^1 can be thought of as measuring the angle without reducing modulo 2π, or modulo 1, in the coordinates on $\mathbb{R}$. Thus, the points t, $t+1$, and $t+2$ in $\mathbb{R}$ all represent the same point in $\S^1$. In the same way, the lift of $f : S^1 \to S^1$ to $F : \mathbb{R} \to \mathbb{R}$ gives the new location without reducing modulo 1. The difference $F(t) - t$ is the amount the point is move around the circle without reducing modulo 1.

p. 50: (L. -9) Explanation: Let $F_\lambda(t) = t + \lambda$ be the rigid rotation. Then the change of angle, $F_\lambda(t)(t) - t = \lambda$ is the same for any point. For an arbitrary homeomorphism of S^1, the change $F(t) - t$ can vary with the point t. The quantity

$$F^n(t) - t = [F^n(t) - F^{n-1}(t)] + [F^{n-1}(t) - F^{n-2}(t)] + \cdots + [F(t) - t]$$

is the total change of angle by the n^{th}-iterate without reducing modulo 1. The average change of angle for one iterate by the first n-iterates is

$$\frac{1}{n}\{F^n(t) - t\} = \frac{1}{n}\{[F^n(t) - F^{n-1}(t)] + [F^{n-1}(t) - F^{n-2}(t)] + \cdots + [F(t) - t]\}.$$

Taking the limit as n goes to infinity, $\lim_{n\to\infty} \frac{1}{n}\{F^n(t) - t\}$ gives the average change of angle for one iterate along the whole orbit. This last limit is used to define the rotation number of the map on the circle.

p. 51 (L. -5) $F^{kp}(t) - t < k[\ldots$

p. 67 (L. -14) $f(I_2) \supset I_2$

p. 69: (L. 13) prove the existence of all the periodic points implied by ...

p. 72 (L. 1-3) First assume there is such a K_0. There is a minimal cycle as in Claim 4 with $2 \le k \le n - 1$. Thus, $I_1 \to I_2 \to \cdots \to I_k \to I_1$ is a cycle of length k, and so there is a periodic point of period k which is less than n. This contradiction implies that the minimal n is 2 in this case.

p. 75 (L. -19, -18) " ... on a sequence in Σ_A^+ gives another sequence in Σ_A^+."

p. 77 (L. -3) "but not eventually positive."

p. 78 (Proof of Lemma 2.5) 'It is clear that $\{\sigma_A^{k+j}(\mathbf{s}^*)\}_{j\ge 0}$ ... "

p. 79 (L. -8) there is an allowable word $\mathbf{w}$ such that

p. 81 (L. -10, -9) Corollary 2.3 and Lemma 3.1

p. 84 (L. 5) dense in $F_\mu[0, 1]$

p. 88 (L. 18) $d(f^k(\mathbf{x}), f^k(\mathbf{p})) \geq \delta$

p. 89 (L. -17) $|T'(x_j)| = 2$

p. 90 (L. 19, 20, 25, 26) add another)

p. 91 (L. 2) $\int_0^1 \lambda(x)\, d\mu(x) =$

p. 92: (Problem 3.7) Hint: Take the double of the map in the previous problem (3.6).

p. 92: (L. -7) $s_n = 1$

p. 96: (L 5) Explanation: The norm of a matrix can be calculated in terms of an eigenvalue of a related matrix. Notice that

$$|A\mathbf{x}|^2 = (A\mathbf{x})^t A\mathbf{x} = \mathbf{x}^t A^t A\mathbf{x}.$$

The maximum of this quantity as $\mathbf{x}$ varies over unit vectors is the square of the norm of A. The matrix A^tA is symmetric and so has real eigenvalues. If λ_1 is the largest eigenvalue with unit eigenvector $\mathbf{v}^1$ then

$$\mathbf{v}_1^t A^t A\mathbf{v}_1 = \mathbf{v}_1^t \lambda_1 \mathbf{v}^1 = \lambda_1.$$

Therefore the norm of A is the square root of the largest eigenvalue of A^tA, $\|A\| = \sqrt{\lambda_1}$.

p. 100 (L. 8) Proposition 3.1

p. 102 (Remark) Remark 3.2

p. 103 (Remark) Remark 3.3

p. 104 (L 15) Evaluate derviative at $t = t_0$.

p. 111 (L. 17) Then, any $\mathbf{x}$ can be written as

p. 113 (L. 14) Define the *stable subspace* (or *stable eigenspace*), unstable subspace (or *unstable eigenspace*), and *center subspace* (or *center eigenspace*) to be

p. 114 (L. 6) $\mathbf{v} \in V^u$ should be $\mathbf{v} \in V^c$: "as $t \to \pm\infty$, so $\mathbf{v} \in V^c$."

p. 116 (L -6) "many different linear contractions" should be "many different linear differential equations"

p. 122 (L. -5, -3) "surround all the eigenvalues of A whose absolute value is less than 1 and is oriented counterclockwise. ... surround all the eigenvalues of A whose absolute value is greater than 1 but"

p. 124 (Lemma 9.7) Let D_k be a block with complex eigenvalues in the Jordan Canonical Form of A given as follows: ... (i) $A_0 = D_k$, (ii) A_1 is a diagonal matrix with real eigenvalues, ... Thus, we have given a curve of matrices from a block in the Jordan Canoical Form corresponding to a complex eigenvalue to a diagonal block with real eigenvalues.

p. 124 (add a Lemma 9.7b) Let A be a matrix with all real eigenvlues. Then, there is a curve of matrices A_t for $0 \leq t \leq 1$, such that (i) $A_0 = A$, (ii) A_1 is diagonalizable (has a basis of eigenvectors), and (iii) the eigenvalues with multiplicities for all the A_t are the same as A.

p. 124 (L 21) Exercises 4.11 and 4.12 ask the reader to prove the following result using Lemmas 9.7, 9.7b, and 9.8.

p. 127 (L. 3) $\lambda_j^2 = \lambda_1^2$

p. 130 (Exercise 4.11a) Hint: Use Lemmas 9.7, 9.7b, and 9.8. Allow for 1's in the off diagonal terms of the Jordan Canonical Form.

p. 134 (L. -7 to -4) Replace with: "If U is a region where $f(\mathbf{x})$ is defined and C^1 and $V \subset U$ is a compact subset, then we can let $K = \sup\{\|Df_{\mathbf{x}}\| : \mathbf{x} \in V\}$. By the Mean Value Theorem,

$$|f(\mathbf{x} - f(\mathbf{y})| \le K|\mathbf{x} - \mathbf{y}|$$

if the line segment from $\mathbf{x}$ to $\mathbf{y}$ is contained in V.

p. 135 (L. 22) Note $L^2(\mathbb{R}^k, \mathbb{R}^n)$ are those maps from $\mathbb{R}^k \times \mathbb{R}^k$ to $\mathbb{R}^n$ which are linear in each factor

p. 139 (L. -10)

$$DF_{(\mathbf{x}_0,\mathbf{y}_0)} = \Big((\frac{\partial f_i}{\partial x_j}(\mathbf{x}_0, \mathbf{y}_0)), (\frac{\partial f_i}{\partial y_{j'}}(\mathbf{x}_0, \mathbf{y}_0)) \Big).$$

p. 143 (Line 7–9) For $\mathbf{x}_0 \in U$ take $b > 0$ such that the closed ball $\bar{B}(\mathbf{x}_0, b) \equiv \{\mathbf{x} : |\mathbf{x} - \mathbf{x}_0| \le b\} \subset U$. The function f is Lipschitz ... for all $\mathbf{x}, \mathbf{y} \in \bar{B}(\mathbf{x}_0, b)$.

p. 153: (L -2) $|e^{At}\mathbf{y}_0|_* \le e^{-tb}|\mathbf{y}_0|_*$

p. 156 (Theorem 5.6) It is not necessary to assume the fixed points are hyperbolic.

p. 172 (Example 8.2) In the case when $r_0 \neq r*$, $r(t)$ should be r^* plus the quantity given.

p. 181: (Theorem 9.1) The region does not have to be simply connected. It should read: "either an open subset of $\mathbb{R}^2$ or $\mathcal{D} = S^2$."

p. 183: (Theorem 9.6(c)) The collection of orbits is countable (finite or infinite). Conti has an example where there is an infinite countable collection of orbits.

p. 186 (L. 10) $B_N = \bigcap_{j=0}^N f^j(\mathbb{E}^u(r) \times \mathbb{E}^s(r))$.

p. 189: (Three lines above Remark 10.1) derivate should be derivative.

p. 191: (L -3) Then, $Dh_{\mathbf{y}} = A_{uu}(\mathbf{q}_s, \mathbf{y})$, which

p. 195: (L 13, 16) W^u_r should be W^s_r.

p. 196: (L 10-12) $\mathbb{E}^u(r)$ should read $\mathbb{E}^s(r)$ and all the σ^u should be σ^s

p. 202: (L 9) Need an extra) at the end of the right hand side.

p. 207: (Exercise 5.16) 8/3 should read -8/3 in the differential equations.

p. 209: (5.27 last line) For $k \ge 1$, prove that f and g_k are not topologically conjugate.

p. 211: (5.40) It should read: "Assume that $\tilde{X}_1(x, a) < 0$, $\tilde{X}_1(x, b) > 0$, and $\tilde{X}_2(x, a) = 0 = \tilde{X}_2(x, b)$ for all x, "

p. 216: (L -18) "from" should be "form"

p. 234 (Exercise 6.5) A *symplectic basis* is a basis of vectors $\{\mathbf{v}^j\}_{j=1}^{2n}$ such that $\omega(\mathbf{v}^j, \mathbf{v}^{j+n}) = 1$ and $\omega(\mathbf{v}^{j+n}, \mathbf{v}^j) = -1$ for $j = 1, \dots n$, and $\omega(\mathbf{v}^j, \mathbf{v}^k) = 0$ for $k \neq j \pm\ n$.

p. 261 Exercise 7.10) 8/3 should read -8/3.

p. 264 (L 17) In the definition of immersion, isomorphism should be injective.

p. 265 (L 18) In this chapter and Chapter 10 ...

p. 269 (Definition) Hyperbolic invariant sets are usually compact; they always can be taken to be closed since the splitting and estimates go over to the closure. Rather than add the assumption of compactness to the definition of a hyperbolic invariant set, we state this hypothesis in the theorems.

p. 271 (Theorem 1.2) ... Let Λ be a compact hyperbolic invariant set. ...

p. 272 (Proposition 1.3) Let Let Λ be a compact hyperbolic invariant set ...

p. 275 (L 27) $a_{s_j,j+1} = 1$ should be $a_{s_j,s_{j+1}} = 1$

p. 276 (Line 8-10) In both the definitions of adjaceny matrix and transition matrix add the conditions that (ii) $\sum_j a_{ij} \geq 1$ for all i, and (iii) $\sum_i a_{ij} \geq 1$ for all j.

p. 277 (L 5) $\gamma(i) = s_i \in S$.

p. 277 (L -6) $S \cap F^{-1}$ should be $S \cap f^{-1}$

p. 279 (Figure 4.2) $F(G)$ should be $f(G)$.

p. 280 (L 14) "three properties" should be "two properties".

p. 289 (L 16) "the full two-sided subshift" should be "the two-sided full shift".

p. 290 (L 6) Λ should be $\Lambda_{\mathbf{q}}$.

p. 293 (L -5) "open set" should read "open neighborhood".

p. 295 (L 4) "In the next subsection" should read "In Subsection 8.4.5"

p. 295 (L -9) "the next subsection" should read Section 6.1"

p. 295 Example 4.1 should be renumbered Example 4.2.

p. 295 Figure 4.9 should refer to the renamed Example 4.2.

p. 298 (L -10) $H^{n_2-1}(U \setminus N_1)$ should be $H_{n_2-1}(U \setminus N_1)$.

p. 299 (L 10) Section 5.5.7 should read Section 5.8.

p. 300 (L10) Subsection 8.4.3 should read Subsection 8.4.5.

p. 302: (L -17) "it" should be "if"

p. 302: (L -10) remove (from $(W^s(p)$.

p. 304: Theorem 4.6 should be 4.7

p. 304: Theorem 4.7 should be 4.8

p. 305 Example 4.2 should be renumbered Example 4.3.

p. 306 (Line 15) It should be $\int_{-\infty}^{\infty} \operatorname{sech}(s)\, \cos(\omega s)\, ds$

p. 307 Example 4.3 should be renumbered Example 4.4.

p. 309 (L 1) remove $\circ$ from "$f_A \circ (\mathbf{x})$", i.e., $f_A(\mathbf{x})$.

p. 309 (L -1) Add an extra ")" to the subscript of $T_{f_A(p)}\mathbb{T}^n$

p. 310 (L -1) f should read f_A. (Also page 311, Lines -3, -9. -11)

p. 315 (L 14-15) We take the images of the interiors because $R_{s_1} \cap f_A^{-1}(R_{s_2})$ does not always equal $\operatorname{cl}(\operatorname{int}(R_{s_1}) \cap f_A^{-1}(\operatorname{int}(R_{s_2})))$but can have extra points whose images are on the boundary of R_{s_2}.

p. 316 (L -2) $W^\sigma(\mathbf{z}', \operatorname{int}(R_k)) = W^\sigma(\mathbf{z}', R_k) \cap \operatorname{int}(R_k)$ for $\sigma = u, s$

p. 318 (L 9) Section 7.3.1 should read Section 8.3.1.

p. 319 (L 5) F_A should be f_A.

p. 320 (L -10) " . . . the eigenvalues of the transition matrix are always plus or minus the eigenvalues of the original matrix A together with possibly 0 and/or roots of unity."

p. 321 (L 1) Theorem 5.8 should be Theorem 5.4.

p. 323 (L 1) Theorem 5.4 should be Theorem 5.5.

p. 323 (L -6) $Df_{\mathbf{p}}|\mathbb{E}^u_{\mathbf{p}} : \mathbb{E}^u_{\mathbf{p}} \to \mathbb{E}^u_{\mathbf{p}}$

p. 324 (L 1) Theorem 5.5 should be Theorem 5.6.

p. 324 Propsition 5.6 should be Propsition 5.7.

p. 324 (L -2, -3) K_i should be K_j.

p. 325 (L 4) $-t^j$ in the numerator should be $(-t)^j$.

p. 325 Propsition 5.7 should be Propsition 5.8.

p. 325 Lemma 5.8 should be Lemma 5.9.

p. 326 (L 7) Lemma 5.9.

p. 326 (L 9) Proposition 5.8 and Proposition 5.7.

p. 326 (L 13) Proposition 5.7 should be Proposition 5.8.

p. 330 (L -2) "interest" should be "intersect"

p. 341 (L 3) In Section 12.2, we show that the tangent lines, $\mathbb{E}^s_{\mathbf{x}} = T_{\mathbf{x}} W^s(\mathbf{p})$, depend in a C^1 fashion on $\mathbf{x}$.

p. 348 (L 4, 8) x should be $|x|$.

p. 354 (L -8) $F'(\theta) = 1 + \epsilon 2\pi k \, \cos(2\pi k\theta)$.

p. 362 (8.13) Let A be an $N \times N$ transition matrix which is irreducible, and $\Sigma_A \subset \Sigma_N$ ...

p. 362 (8.15a) It should be T-allowable words $\mathbf{w}$ of length k not $k+1$.

p. 363 (8.20) This exercise should refer to the renumbered Example 4.2 (Section 8.4.2). This example appears on page 295, and should be renumbered as noted above.

p. 367 (8.43) The assuption on $\mathbf{x}$ should be that it is ω-recurrent, $\mathbf{x} \in \omega(\mathbf{x})$, and not that it is chain recurrent.

p. 368 (8.48(b)) f should be g.

p. 371 The proof of Theorem 1.2 should read as follows: The points of the orbits considered for $r(n,\delta,f^k)$ constitute a subset of those considered for $r(nk,\delta,f)$,

$$\{f^{ki}(\mathbf{y})\colon 0 \le i < n\} \subset \{f^i(\mathbf{y})\colon 0 \le i < nk\},$$

so $d_{f,nk}(\mathbf{x},\mathbf{y}) \ge d_{f^k,n}(\mathbf{x},\mathbf{y})$, and any (n,δ)-separated set for f^k is also an (nk,δ)-separated set for f, and we have that $r(n,\delta,f^k) \le r(nk,\delta,f)$. By uniform continuity, given $\epsilon > 0$, there is $\delta_\epsilon > 0$ such that if $d(\mathbf{x},\mathbf{y}) \le \delta_\epsilon$, then $d(f^j(\mathbf{x}), f^j(\mathbf{y})) \le \epsilon$ for $0 \le j < k$. So, if $d_{f^k,n}(\mathbf{x},\mathbf{y}) \le \delta_\epsilon$ then $d_{f,nk}(\mathbf{x},\mathbf{y}) \le \epsilon$, or $d_{f,nk}(\mathbf{x},\mathbf{y}) > \epsilon$ then $d_{f^k,n}(\mathbf{x},\mathbf{y}) > \delta_\epsilon$. Therefore, any (nk,ϵ)-separated set for f is also a (n,δ_ϵ)-separated set for f^k, or $r(n,\delta_\epsilon,f^k) \ge r(nk,\epsilon,f)$, where δ_ϵ is uniform in n. Combining these two inequalities,

$$\frac{1}{n}\,\log(r(nk,\epsilon,f)) \le \frac{1}{n}\,\log(r(n,\delta_\epsilon,f^k)) \le \frac{1}{n}\,\log(r(nk,\delta_\epsilon,f)),$$

and taking the limits in n and then ϵ (so $\delta_\epsilon \le \epsilon$ also goes to zero)

$$\begin{aligned} k\,h(\epsilon,f) &\le h(\delta_\epsilon,f^k) \le k\,h(\delta_\epsilon,f),\\ k\,h(f) &\le h(f^k) \le k\,h(f). \end{aligned}$$

This proves the theorem.

p. 373 (L 4, Remark 1.5) Theorem 1.6 should be Proposition 1.6.

p. 373 (Remark 1.7) Remove the comment "(because if ... infinity)".

p. 375 (L 11) y_j should be $H^{i-j}(y_j)$

p. 376 (Theorem 1.7) The assumption that X and Y are compact has to be added to part (b), or the assumption moved to the general assumptions of the theorem.

p. 376 (L 6) $d'(k(\mathbf{x}_1), k(\mathbf{x}_2))$

p. 378 (L -3) "where $B_j = A_{j,j+1} \cdots A_{k-1,k} A_{k,1} \cdots A_{j-1,j}$" i.e., $A_{k-1,k}A_{k,1}$ and not $A_{n-1,n}A_{n,1}$

p. 379 (L -3) $K \subset X$

p. 379 (L -1) $\#(E_{span}(m, \epsilon, K)) = r_{span}(m, \epsilon, K, f)$.

p. 380 (L 3-4) Again, we let $E_{sep}(m, \epsilon, K)$ be a maximal (m, ϵ)-separated set for K, so $\#(E_{sep}(m, \epsilon, K)) = r_{sep}(m, \epsilon, K, f)$.

p. 380 (L -9) $h_{sep}(K, f) = h_{span}(K, f)$

p. 380 (L -6 to -4) $\dots$ is bounded by $N^n_{\epsilon/2}$. (There cannot be two orbits with $d_{n,f}(\mathbf{x}, \mathbf{y}) \geq \epsilon$ and $f^j(\mathbf{x})$ and $f^j(\mathbf{y})$ in the same $\epsilon/2$-balls for $0 \leq j < n$.) Therefore, $r_{sep}(n, \epsilon, K, f) \leq N^n_{\epsilon/2}$, and $h_{sep}(\epsilon, K, f) \leq \log(N_{\epsilon/2}) < \infty$.

p. 381 (L -16) $E_m(\epsilon, \Omega)$ should be $E_{span}(m, \epsilon, \Omega)$.

p. 384 (L 7 - 12) The obvious attempts at proofs do not work. Using the uniform continuity of k, it can be shown that given $\epsilon > 0$ there is a $\delta > 0$ such that if $E_{sep}(n, \epsilon, f) \subset Y$ is (n, ϵ)-separated for f, then $k^{-1}(E_{sep}(n, \epsilon, f))$ is (n, δ)-separated for F. However, $k^{-1}(E_{sep}(n, \epsilon, f))$ is not necessarily the maximal (n, δ)-separated set for F, so this fact does not give an upper bound for $r_{sep}(n, \delta, F)$ in terms of $r_{sep}(n, \epsilon, f)$.

p. 385 (L 5) $r_{span}(n, \beta, F)$ should be $r_{span}(n, \beta, f)$.

p. 385 (L 16) $0 \leq s \leq \ell$ should be $0 \leq s < \ell$.

p. 387 (L 10) $f|\Lambda'$ should be $h(f|\Lambda')$.

p. 387 (L 13) $\#(\text{Fix}(f^k))$ should be $\#(\text{Fix}(f^n))$

p. 391 (Line 6) The way to calculate the limits of the wedge product is to start with an orthonormal basis $\{\mathbf{v}^{0,1}, \dots, \mathbf{v}^{0,m}\}$ of tangent vectors at $\mathbf{x}^0 = \mathbf{x}$. Let $\mathbf{x}^k = f^k(\mathbf{x})$. Assume by induction that we have defined an orthonormal basis $\{\mathbf{v}^{k-1,1}, \dots, \mathbf{v}^{k-1,m}\}$ at $\mathbf{x}^{k-1}$. Applying the derivative at x^{k-1}, let $\mathbf{w}^{k,j} = Df_{\mathbf{x}^{k-1}}\mathbf{v}^{k-1,j}$ be the image vectors. Apply the Gram-Schmidt process to construct a basis of perpendicular vectors:

$$\begin{aligned} \mathbf{z}^{k,m} &= \mathbf{w}^{k,m} \\ \mathbf{z}^{k,m-1} &= \mathbf{w}^{k,m-1} - \frac{\mathbf{w}^{k,m-1} \cdot \mathbf{z}^{k,m}}{|\mathbf{z}^{k,m}|^2}\mathbf{z}^{k,m} \\ \mathbf{z}^{k,j} &= \mathbf{w}^{k,j} - \sum_{i=j+1}^{m} \frac{\mathbf{w}^{k,j} \cdot \mathbf{z}^{k,i}}{|\mathbf{z}^{k,i}|^2}\mathbf{z}^{k,i} \qquad \text{for } 1 \leq j \leq m-1. \end{aligned}$$

We get an orthonormal basis of vectors at x^k by letting

$$\mathbf{v}^{k,j} = \frac{\mathbf{z}^{k,j}}{|\mathbf{z}^{k,j}|}.$$

This completes the induction process. The multiplicative factor of the j^{th}-vector is

$$r_j^{(k)} = |\mathbf{w}^{1,j}| \cdots |\mathbf{w}^{k,j}|.$$

The volume of the parallelograms spanned by $\{\mathbf{z}^{k,m-j+1}, \dots, \mathbf{z}^{k,m}\}$ is the same as that spanned by the $\{\mathbf{w}^{k,m-j+1}, \dots, \mathbf{w}^{k,m}\}$, which is $r^{(k)}_{m-j+1} \cdots r^{(k)}_m$. Thus the growth rate of this volume as k goes to infinity is

$$\begin{aligned} \lambda_{m-j+1} + \cdots \lambda_m &= \lim_{k\to\infty} \frac{1}{k} \log(r^{(k)}_{m-j+1} \cdots r^{(k)}_m) \\ &= \lim_{k\to\infty} \frac{1}{k} \log(r^{(k)}_{m-j+1}) + \cdots \lim_{k\to\infty} \frac{1}{k} \log(r^{(k)}_m), \end{aligned}$$

and

$$\lambda_{m-j+1} = \lim_{k\to\infty} \frac{1}{k} \log(r^{(k)}_{m-j+1}) = \lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} \log(|\mathbf{w}^{i,m-j+1}|).$$

p. 393 (L 13) "The only situation" should be "One situation"

p. 394 (L 18-20) Should be "Thus, for a compact submanifold A of dimension d and $0 \le p < d < q$, the limit, as the size of boxes tends to zero, of the p-dimensional volume of boxes which cover A is infinite, the limit of the q-dimensional volume of boxes which cover A is zero, and the limit of the d-dimensional volume of the boxes which cover A is a finite number."

p. 399 (Ex. 9.15(a)) "of radius" is repeated.

p. 401 (Ex. 9.28) It should read $f(t, z) = (g(t), \beta\, z + \frac{1}{2}\, e^{2\pi t i})$.
(a) Prove for $0 < \beta < 1/(2\sqrt{2})$, ...
(b) " Also prove for the correct choice ... "

p. 404 (L-9) The concept of a trapping region is related to an isolating set but is not the same thing. Therefore the comment "(or *isolating neighborhood* by Conley)" should be removed.

p. 405 (L -5 & -4) This should read "By taking the intersection of these sets, $\mathcal{P} \subset \bigcap_{0\le j\le 5} A_j \cup A^*_j = \{\mathbf{p}_j : 0 \le j \le 3\}$. By Remark 1.4, ... "

p. 407 (L. -16) "for part (b)" should be "for part (a)"

p. 408 (L. -14) "absolutely convergent" should read "uniformly convergent"

p. 409 (L. 2 & 3) The + sign should be - on both lines.

p. 411: (L -18) "we do not explicity emphasize the fact that $D(\exp_{\mathbf{p}})_{\mathbf{v_p}}$ is different from the identity."

p. 412: (L -17) $D(\exp_{f(\mathbf{p})})_{\mathbf{0_p}} = id$ should be $D(\exp_{f(\mathbf{p})})_{\mathbf{0}_{f(\mathbf{p})}} = id$.

p. 413: (L 5) $\tilde{W}^s_r(\mathbf{p}) = \bigcap_{j=0}^{\infty} (F_{f^{j-1}(\mathbf{p})} \circ F_{f^{j-2}(\mathbf{p})} \circ \cdots \circ F_{\mathbf{p}})^{-1}(\mathcal{B}_{f^j(\mathbf{p})}(r))$

p. 415: (Theorem 3.1) (The second half of this theorem should read as follows.) Moreover, there is an $\epsilon_0 > 0$ such that if $0 < \epsilon \le \epsilon_0$, $j_1 = -\infty$, and $j_2 = \infty$ for the δ-chain, then $\mathbf{y}$ is unique. If $0 < \epsilon \le \epsilon_0$, $j_2 = -j_1 = \infty$, and Λ is an isolated invariant set (or has a local product structure), then the unique point $\mathbf{y} \in \Lambda$.

p. 417: (Example 3.3) Let $\Lambda \subset \Sigma_2$ be the subshift of Example 3.1. Let $\mathbf{t} \in \Sigma_2$ be the sequence with $t_i = 1$ for $i < 0$, $t_0 = 2$, $t_1 = 1$, $t_2 = 2$, $t_3 = t_4 = t_5 = 1$, $t_6 = 2$, etc. After each 2 in the sequence, there are odd number of ones, with each time two more ones than the previous time. The point $\mathbf{t}$ is in $W^s(\Lambda)$ because for any $\epsilon > 0$, for any sufficiently large n, $\sigma^n(\mathbf{t})$ is within ϵ of the orbit of $\bar{1}.2\bar{1} \in \Lambda$. On the other hand, $\mathbf{t}$ is not in the stable manifold of a single point in Λ.

p. 422 (L 1) In fact, if Λ is a connected hyperbolic attracting set for a diffeomorphism f and the periodic points are dense in Λ, then f is topologically transitive on Λ.

p. 423 (L 9) $f|Lam_i$ should be $f|\Lambda_i$

p. 423: In the proof of Theorem 5.4, if we assume that $\mathcal{R}(f)$ is hyperbolic, then it is possible to take the chain componenets rather than the sets $\mathrm{cl}(H_{\mathbf{p}})$ in the decomposition.

p. 424: (L -9) $W^u(\mathcal{O}(\mathbf{p}))$ should be $W^s(\mathcal{O}(\mathbf{p}))$,

p. 428: (L -12) s^*_i should be s^*_j

p. 435: (L9) " . . . the set of homeomorphisms are not open, so a small perturbation does not have to be a homeomorphism. Therefore, . . . "

p. 444 (L4-6) "A *fundamental domain for the stable manifold of* Λ is a closed set $D^s \subset W^s(\Lambda) \setminus \Lambda$ such that there exists a set $D^{s\prime}$ with $D^s = \text{cl}(D^{s\prime})$ and $f^j(D^{s\prime}) \cap D^{s\prime} = \emptyset$ for all integers $j \neq 0$, and $\bigcup_{j\in\mathbb{Z}} f^j(D^s) = W^s(\Lambda) \setminus \Lambda$."

p. 446 (Ex. 10.22) Assume $\Omega(f) = M$.

p. 449: (L -5) $\text{Per}(k, f)$ should be $\text{Per}(n, f)$.

p. 450: (L -22) $\mathcal{H}(X)$ should be $\mathcal{H}(M, \mathfrak{X})$, and $\mathcal{KS}(X)$ should be $\mathcal{KS}(M, \mathfrak{X})$

p. 450: (L -20) γ should be γ_2.

p. 456: (L -16) $D^{Lm}(r)$ should be $D^{Im}(r)$

p. 456: (L -6) $K_{i,1}$ should be K_i

p. 458: (L 5) n should be m

p. 458: (Lemma 3.3) ρ_1 should be ρ_n.

p. 459: (L -14) $\mathcal{R}$ should be $\mathbb{R}$.

p. 464: (L 3-4) If $\mathcal{R}(f)$ is also hyperbolic, then f also satisfies the transversality condition with respect to $\mathcal{R}(f)$, by Theorem 4.3 (or Exercise 11.11(c)),

p. 464: (L 16-17) "the the" should be "the"

p. 464: (L 17 & 24) Section 8.7 should be Section 8.8.

p. 466: (L 3) f should be f' in definition of $\mathcal{N}_2$.

p. 466: (Exercise 11.4) "j periodic sinks" and "j periodic sources" should be "k periodic sinks" and "k periodic sources".

p. 467: (Exercise 11.9) "Given" should be "Give".

p. 469: (L 4) "Chapters V and IX" should be "Chapters V and X".

p. 473: (Remark 1.3) Hurder and Katok proved a result like Theorem 1.2 with the assumptions as stated in the theorem. A. Wilkinson proved the result for C^α in her 1995 thesis from the University of California at Berkeley. (Erg. Theory and Dyn. Sys. **18** (1998), 1545 − 1587.)

p. 475: (L -4) $\mathbf{L}(T_\mathbf{x}, Y)$ should be $\mathbf{L}(T_\mathbf{x}X, Y)$.

index: All references to Chapters VII - XII in the Index are off by two pages. For example the reference to "homoclinic point" is page 285, but the correct reference is 287.

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