ADDITIONAL PROBLEMS FOR DYNAMICAL SYSTEMS: STABILITY, SYMBOLIC DYNAMICS, AND CHAOS

by Clark Robinson

Also see the list of Extra Problems for the Second Edition of the book.

p. 202 Additions to problem. 5.4.

(b) Assume that g is Lipschitz. Prove that σ is a Lipschitz function.

(c) Assume that Λ is an open subset of a Euclidean space $\mathbb{R}^k$, Y is a closed ball in $\mathbb{R}^n$, and g is a C^r function on $\Lambda \times \mathrm{int}(Y)$. Apply the Implicit Function Theorem to $G(\mathbf{x}, \mathbf{y}) = g(\mathbf{x}, \mathbf{y}) - \mathbf{y}$ to prove that σ is a C^r function.

p. 328 Extra problem for Anosov Diffeomorphisms: Let f_A be a hyperbolic toral automorphism on $\mathbb{T}^n$ with lift L_A to $\mathbb{R}^n$. Let g be a map of $\mathbb{T}^n$ which is homotopic to f_A. Use Problem 5.30 to prove that g is semi-conjugate to f_A.

p. 329 Improved wording for 7.25. (A horseshoe as a subsystems of a hyperbolic toral automorphism.) Let $f_{A_2} : \mathbb{T}^2 \to \mathbb{T}^2$ be the diffeomorphism induced by the matrix

$$A_2 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

discussed in Example 5.4. Let R_{1a} be the rectangle used in the Markov partition for this diffeomorphism. Let $g = f_{A_2}^2$ and

$$\Lambda = \bigcap_{j=-\infty}^{\infty} g^j(R_{1a}).$$

Prove that $g : \Lambda \to \Lambda$ is topologically conjugate to the two-sided full two-shift $\sigma : \Sigma_2 \to \Sigma_2$. Hint: R_{1a} plays the role that S played in the construction of the geometric horseshoe. Prove that $g(R_{1a}) \cap R_{1a}$ is made up of two disjoint rectangles. (These rectangles are similar to V_1 and V_2 in the geometric horseshoe.) Looking at the transition matrix for the Markov partition for f_{A_2} may help.

p. 332 Extra problem. 7.47. Assume $f : \mathbb{R}^2 \to \mathbb{R}^2$ is a diffeomorphism with a compact invariant set Λ. Assume Λ satisfies all the conditions for a hyperbolic structure except possibly the continuity of the splitting. Assume the dimension of each of the subspaces in the splitting has dimension one. Prove that the splitting is continuous. Hint: Consider the argument in the separate addition to page 195.

p. 332 Extra problem. 7.48. Assume $f : \mathbb{R}^2 \to \mathbb{R}^2$ is a diffeomorphism with a compact hyperbolic invariant set Λ. Prove that if V is a small enough neighborhood of Λ, then the maximal invariant set in V,

$$\Lambda_V \equiv \bigcap_{n \in \mathbb{Z}} f^n(V),$$

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has a hyperbolic structure. Hint: See the proof of Theorem 4.5.

p. 332 Extra problem. 7.49. Let $f : \mathbb{R} \to \mathbb{R}$ be C^1 function Assume that $f(0) = 0$ and $f'(0) > 1$. Assume that $q \neq 0$ is a point such that (i) $f(q) = 0$, (ii) there is a choice of a backward orbit $\{q_k\}_{k \le 0}$ with $q_0 = q$, $f(q_{k-1}) = q_k$ for $k \le 0$, $f'(q_k) \neq 0$ for $k \le 0$, and q_k converges to 0 as k goes to $-\infty$, i.e., $q \in W^u(0)$. Prove that there is an invariant set containing q and 0 which is conjugate to a one-sided subshift of finite type. Hint: Since f is not a diffeomrophism, Theorem 4.5 does not apply. Find intervals which work correctly as the boxes do in the proof of Theorem 4.5.

p. 332 Extra problem. 7.50. Assume the flow ϕ^t is gradient-like with Liapunov function L.

a. Prove that $\Omega(\phi^t) = \mathrm{Fix}(\phi^t)$.

b. Assume that $L(\mathrm{Fix}(\phi^t))$ is a totally diconnected subset of $\mathbb{R}$. Prove that $\mathcal{R}((\phi^t) = \mathrm{Fix}(\phi^t)$.

p. 332 Extra problem. 7.51. Consider the map given in polar coordinates by

$$F\begin{pmatrix} \theta \\ r \end{pmatrix} = \begin{pmatrix} 8\,\theta - \dfrac{\pi}{8} \\ 2 + \dfrac{1}{16}\,r + \dfrac{1}{16}\,\sin(\theta) \end{pmatrix}$$

for $0 \le \theta \le \dfrac{\pi}{2}$ and $1 \le r \le 3$. (Notice this definition is only for part of the plane.) Let

$$\begin{aligned} V_L &= \{(\theta, r)\colon 0 \le \theta \le 3\,\pi/32,\ 1 \le r \le 3\} \\ V_R &= \{(\theta, r)\colon \pi/4 \le \theta \le 11\,\pi/32,\ 1 \le r \le 3\}. \end{aligned}$$

Prove that the maximal invariant set in $V_L \cup V_R$ is hyperbolic.

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