ADDITIONAL PROBLEMS FOR DYNAMICAL SYSTEMS: STABILITY, SYMBOLIC DYNAMICS, AND CHAOS SECOND EDITION

by Clark Robinson

p. 63 Extra Problem 2.32. Assume F is a lift of f. Prove that F^q is a lift of f^q.

p. 63 Extra Problem 2.33. Assume that F_1 and F_1 are lifts of f. Prove that there is an integer p such that $F_2(x) = F_1(x) + p$ for all x.

p. 63 Extra Problem 2.34. Assume that f on X and g on Y are semi-conjuage via the map h, $h \circ f = g \circ h$.

(a) If p is a periodic point for f with period n, prove that $h(p)$ is a periodic point for g whose period divides n. If f and g are conjuage, prove the periods of p and $h(p)$ are the same.
(b) Assume p is an asymptotically stable fixed point for f. Prove that $h(p)$ is an asymptotically stable fixed point for f.
(c) Assume that f and g are conjugate and p is a fixed point for f with $f'(p) > 0$. Prove that $h(p)$ is a fixed point with $g'(h(p)) \geq 0$.
(d) Prove that if f is topologically transitive on X, then g is topologically transitive on Y.
(e) Prove that if f is topologically mixing on X, then g is topologically mixing on Y.

p. 63 Extra Problem 2.35. Let $f : S^1 \to S^1$ be an orientation preserving homeomorphism. Let $h : S^1 \to S^1$ be an orientation reversing homeomorphism. Prove that the rotation number of $h^{-1} \circ f \circ h$ is equal to the negative of the rotation number of f mod 1,

$$\rho(h^{-1} \circ f \circ h) = -\rho(f) \mod 1.$$

p. 94 Extra Problem 3.24. Define the map

$$f(x) = \begin{cases} a\,x + c & \text{if } 0 \leq x \leq c \\ a - a\,x & \text{if } c \leq x \leq 1, \end{cases}$$

where $c = \dfrac{1}{1+a}$ and $a^2 = a + 1$ are chosen so that $f(c) = 1$ using both definitions. Notice that $a = \dfrac{\sqrt{5}+1}{2} > 1$ is the golden mean.

(a) Draw the graph of f.
(b) Using the partition $I_1 = [0, c]$ and $I_2 = [c, 1]$ what is the transition matrix A for this partition.
(c) Prove that for any allowable sequence $\mathbf{s} \in \Sigma_A^+$, there is a unique point $h(\mathbf{s})$ such that $f^j(h(\mathbf{s})) \in I_{s_j}$.

(d) Prove that σ_A on Σ_A^+ is semi-conjugate to f on $[0,1]$. (Any point which goes through c has two choices of symbols.)
(e) Prove that f is topologically transitive on $[0,1]$.

p. 213 Extra Problem 5.47. Let $F_a : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by the formula

$$f_a(x,y) = (1 - ax^2 + y, x)$$

where $a > 0$ is a parameter.
(a) Final all the fixed points and points of period two.
(b) Find all values of the parameter a for which the points of period two are saddles.

p. 364 Extra problem on horseshoes ($8.23\frac{1}{2}$) Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a diffeomorphism with two hyperbolic saddle fixed points $\mathbf{p}_1$ and $\mathbf{p}_2$. Assume there is a transverse heteroclinic point $\mathbf{q}_1$ in $W^u(\mathbf{p}_1) \cap W^s(\mathbf{p}_2)$ and another transverse heteroclinic point $\mathbf{q}_2$ in $W^u(\mathbf{p}_2) \cap W^s(\mathbf{p}_1)$. Discuss the correct form of the subshift of finite type which is conjugate to the hyperbolic invariant set which contains $\mathcal{O}(\mathbf{q}_1) \cup \mathcal{O}(\mathbf{q}_2) \cup \{\, \mathbf{p}_1,\, \mathbf{p}_2 \,\}$.

p. 401 Extra Problem 9.33. Let $S = \{0\} \cup \{\, 1/k^p : k \text{ is a positive integer } \}$. Prove that $\dim_{\mathrm{b}}(S) = 1/(p+1)$ and $\dim_H(S) = 0$.

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