

Index

- C^1 15, 134
- C^r 15, 136
- $C^r(M, N)$ 263
- $\text{Cont}(n, \mathbb{R})$ 123
- $\text{Diff}^r(M)$ 263
- $\text{Diff}^r(\mathbb{R}^k)$ 136
- \mathbb{E}^c 113, 159
- \mathbb{E}^s 113, 159
- \mathbb{E}_p^s 267, 389
- \mathbb{E}^u 113, 159
- \mathbb{E}_p^u 267, 389
- $\text{Fix}(f)$ 15
- $\mathbf{L}(\mathbb{R}^k, \mathbb{R}^n)$ 96
- $\text{Lip}(f)$ 134
- $m(A)$ 96, 140, 161
- \mathbb{N} 30
- $\text{Per}(f)$ 448
- $\text{Per}(n, f)$ 15, 448
- $\text{Per}_h(f)$ 448
- $\text{Per}_h(k, f)$ 448
- $\mathfrak{X}^r(M)$ 266
- α Cantor set 27
- α -limit point 23, 148
- α -limit set 23, 148
- α -recurrent 25, 151
- accessible points 306
- adapted metric 184
- adapted norm 110, 118, 153, 159, 184, 268
- adding machine 92
- adjacency matrix 274
- admissible metric 468
- affine conjugacy 42
- Andronov-Hopf bifurcation for diffeomorphisms 255
- Andronov-Hopf bifurcation for flows 249
- Anosov Closing Lemma 416
- Anosov diffeomorphism 306, 475
- asymptotic 16
- asymptotically stable 16, 108, 158
- attracting 16, 108, 112, 152, 158, 170
- attracting fixed point 118
- attracting set 324, 402
- attractor 324
- attractor of Lorenz type 351
- Axiom A 417
- backward orbit 16
- Baker's map 61
- Baker's map, generalized 398
- base space 467
- basic sets 421
- basin boundary 305
- basin of attraction 152, 158, 207, 305
- Benedicks and Carleson 341
- Birkhoff center 25, 151
- Birkhoff Ergodic Theorem 89, 272, 388
- Birkhoff normal form 226
- Birkhoff Transitivity Theorem 271
- Borel measure 271
- Borel probability measure 386
- Borel sets 386
- bounded continuous maps 161
- bounded linear map 161
- box dimension 392
- branched manifold 336, 339, 348
- Brusselator 258
- bump functions 166
- bundle map 467
- C^0 -sup-norm 143
- C^r -conjugacy 42
- C^r -diffeomorphism 136
- C^r -distance between functions 45
- C^r Section Theorem 470
- C^r structurally stable 264
- C^r topology 264
- C^r topology on vector fields 266
- Cantor set 27
- capacity dimension 392

- center 106, 152
 center eigenspace 113, 122
 center manifold 201
 center-stable manifold 200
 center-unstable manifold 200
 chain components 26, 151, 402
 chain equivalent 26
 chain limit set 25, 402
 chain recurrent set 26, 151, 402
 chain transitive 26
 chaos 86, 389
 chaotic attractor 325
 chaotic map 86
 characteristic multipliers 170
 closed orbit 148, 168
 cobweb plot 18
 codimension 451
 cones 187, 189, 282, 330
 conjugacy 41
 conjugate flows 115
 conjugate maps 41
 Conley's Fundamental Theorem 402
 continuation of a fixed point 160
 continuity of solutions with respect to
 initial conditions 145
 continuous splitting 267
 continuously differentiable 15, 134, 136
 contracting linear map 118
 contraction mapping 141
 Contraction Mapping Theorem 141
 coordinate chart 261
 covering estimate 140
 critical point 13
 cross section to a flow 168
 cycle 354, 423

 DA-diffeomorphism 332
 Denjoy's Example 56
 Denjoy's Theorem 56
 derivative 134
 diffeomorphism 15, 136, 263
 differentiable 134, 136
 differentiable linearization 156, 160
 differentiable map between manifolds
 263
 differentiably conjugacy 42
 differentiably conjugate 366
 differential equation on a manifold 266
 dimension, box 392
 dimension, capacity 392
 dimension, Hausdorff 396

 dimension, topological 326
 disk bundle 468
 double of a map 73
 doubling map (squaring map) 43, 369
 dynamically defined 396

 ϵ -chain 25, 151
 ϵ -shadow 62, 412
 edge subshift 275
 eigenvalues of a periodic orbit 170
 embedded submanifold 262
 embedding 262
 entropy, topological 368
 equilibrium 148
 equivalent flows 115
 ergodic 272, 386
 eventually periodic 15
 eventually positive matrix 76, 125
 exact symplectic 225
 existence of solutions of differential
 equations 143
 expanding attractor 326
 expanding linear map 118
 expansion 112
 expansive 85, 416

 f -covers 66
 family of quadratic maps 20
 Feigenbaum constant 82
 fiber 467
 fiber contraction 467
 filtration 439
 first return map 168
 First Variation Equation 147
 fixed point 15, 148, 158
 flip bifurcation 244
 Floquet theory 202
 flow 142
 flow box coordinates 206
 flow conjugate 205
 flow equivalent 115, 205
 flow expansiveness 438
 flow of a vector field 266
 focus 107
 forward orbit 16
 Frechet derivative 134
 full n -shift 273
 fundamental domain 45, 46, 47, 49,
 116, 119, 442
 fundamental matrix solution 102

- generalized Baker's map 398
 generalized eigenvector 96
 generalized tent map 59
 generating function 224
 geometric horseshoe 275
 geometric model of Lorenz equations 345
 global stable manifold 185
 global stable manifolds of points 269
 global unstable manifold 185
 global unstable manifolds of points 269
 gradient flow 355
 gradient-like 357
 gradient-like vector fields 402
 gradient vector field 355, 402
 graph transform 185, 468
 Gronwall's inequality 144
 Hamiltonian differential equations 215
 Hamiltonian matrix 219
 Hamiltonian vector field 216
 Hartman-Grobman Theorem 154, 155, 159, 173
 Hausdorff dimension 396
 Hausdorff metric 450
 Hénon attractor 339
 Hénon map 6, 257, 281, 339
 heteroclinically related 418
 higher derivatives 136
 Hölder 135
 homeomorphism 15
 homoclinic 218
 homoclinic point 285
 homoclinically related 418
 homology zeta function 322
 Hopf bifurcation for diffeomorphisms 255
 Hopf bifurcation for flows 249
 horseshoe 6, 275, 285
 h-related 418
 hyperbolic 113, 122, 152, 159, 170, 183
 hyperbolic estimates 190
 hyperbolic fixed point 152
 hyperbolic invariant set 267, 270
 hyperbolic linear differential equation 113
 hyperbolic linear map 122, 183
 hyperbolic periodic point 159
 hyperbolic splitting 113
 hyperbolic structure 267, 270
 hyperbolic toral automorphisms 307
 immersed submanifold 262
 immersion 262
 Implicit Function Theorem 137, 138
 In Phase Theorem 415
 Inclination Lemma 203
 index of fixed point 358
 infinitesimally symplectic 219
 Intermediate Value Theorem 14
 invariant measure 84, 386
 invariant section 467
 Invariant Section Theorem 468
 invariant set 23
 invariant splitting, differentiability 475
 Inverse Function Theorem 140
 inverse limit 332
 irreducible 273
 irreducible matrix 76, 125
 isoclines 175, 208
 isolated invariant set 288, 412
 isolating neighborhood 402, 412
 itinerary map 39, 279, 313
 Jakobson 84, 342
 Jordan canonical form 96
 k-cycle 354, 423
 knots 351
 Kupka-Smale Theorem 448
 Lambda Lemma 203
 least period 15
 Lefschetz Fixed Point Theorem 321
 Lefschetz index of a fixed point 321
 Lefschetz number of a diffeomorphism 321
 Liapunov exponents 86, 88, 386
 Liapunov function 157, 357, 402
 Liapunov stable 16, 108, 158
 Lienard equations 174
 lift 50
 limit cycle 149, 181
 limit set 25, 151, 418
 linear conjugacy 42
 linear contraction 118
 linear expansion 118
 linear sink 118
 linear source 118
 Liouville's formula 80, 103, 180, 210
 Lipschitz 134
 local product structure 314, 420, 424

- local stable manifold 184
 local unstable manifold 184
 locally eventually onto 349
 Lorenz attractor 345
 Lorenz equations 8, 342
 L-stable 16

 manifold 261
 Markov chain, topological 75
 Markov partition 314, 424
 Markov property for ambient boxes 279, 291
 maximal interval of definition of the solution 145
 maximal invariant set 286, 412
 Mean Value Theorem 14
 measure 386
 Melnikov function 302
 Melnikov method 7, 302
 middle- α Cantor set 27
 middle-third Cantor 28
 minimal set 25
 minimum norm 96, 140, 161, 190
 mixing 40
 monotonically decreasing 58
 monotonically increasing 58
 Morse inequalities 358
 Morse-Smale diffeomorphism or flow 352
 Morse-Smale flow 355
 Multiplicative Ergodic Theorem 386
 multiplicatively independent 229

 (n, ϵ) -separated 368
 (n, ϵ) -spanning set 377
 negatively invariant set 23
 no cycle property 423
 node 107
 nondegenerate critical point 13, 356
 nonhomogeneous linear equations 117
 nonnegative matrix 125
 nonuniformly hyperbolic structure 385, 389
 nonwandering 151
 nonwandering point 25
 nonwandering set 25
 normal bundle 479
 normal bundle of a closed orbit 173
 normally contracting 479
 normally contracting manifold, persistence 479

 normally hyperbolic 479
 nowhere dense 26
 n -shift 38, 273
 n -sphere 262

 ω -limit point 23, 148
 ω -limit set 23, 148
 ω -recurrent 25, 151
 Ω -Stability Theorem 439
 Ω -stable 48, 439
 one-sided shift 38
 openness of Anosov diffeomorphisms 434
 operator norm 96, 160
 orbit 16, 146
 orbitally Liapunov stable 171
 orientation preserving, reversing 119
 overflowing 467

 partial ordering of basic sets 423
 past history 184
 perfect 27
 period 15, 148, 158
 period doubling bifurcation 244
 period doubling route to chaos 82
 periodic attractor 170
 periodic orbit 15, 148, 168
 periodic point 15, 148, 158, 168
 periodic repeller 170
 periodic sink 170
 periodic source 170
 Perron method for stable manifolds 186
 Perron-Frobenius Theorem 125, 375
 persistence of a fixed point 160
 phase portrait 2, 104, 150
 phase space 104
 pitchfork bifurcation 257
 Plykin attractor 337
 Poincaré map 168
 Poincaré metric 36
 Poincaré norm 36
 Poincaré Recurrence Theorem 359
 Poincaré-Bendixson Theorem 181
 positive matrix 76, 125
 positively invariant set 23
 predator-prey 180
 proper 166

 quadratic map 20
 quasi-periodic 231

- \mathcal{R} -stable 439
 r -continuously differentiable 136
 ρ -distance 34
rectangle (Markov partition) 314, 424
recurrent 25, 151
repelling 16, 112, 152, 159, 170
repelling fixed point 118
repelling set 402
residual subset 271
Riemannian metric 266
Riemannian norm 266
rigid rotation of S^1 51
rooftop map 43
rotation number 50
rotation of S^1 51
- saddle 113, 152, 159
saddle linear differential equation 113
saddle-node bifurcation 236
Schwarz Lemma 35
second derivative 135
sectors 187
semi-conjugacy 41
semi-stability of Anosov Diffeomorphisms 433
semi-stable 433
sensitive dependence on initial conditions 85, 416
separated set 368
shadow 62, 412
Sharkovskii ordering 67
Sharkovskii's Theorem 68
shift space 38, 273, 315
Sinai-Ruelle-Bowen measure 391
sink 16, 112, 152, 159, 170
Smale horseshoe 275, 285
solenoid 326
source 16, 112, 152, 159, 170
spanning set 377
Spectral Decomposition Theorem 420
spectrum of a linear map 183
sphere 262
spiral 107
squaring map (doubling map) 369
SRB measure 391
stability of an hyperbolic invariant set 435
stable bundle of a periodic orbit 202
stable cones 189
stable disk 188
stable eigenspace 113, 122
- stable focus 107
stable manifold 185
stable manifold of an invariant set 415
Stable Manifold Theorem 184, 200
Stable Manifold Theorem for a hyperbolic set 269, 409
stable node 107
stable set 16
stable spiral 107
stable subbundle 267
stable subspaces 184
stair step method 18
standard map 225
Stefan cycle 69
Stefan transition graph 69
Sternberg linearization 156, 160
strongly gradient-like 402
structural stability 45, 264
structural stability, necessary conditions 459
structural stability, nondensity 462
structural stability of Anosov diffeomorphisms 434
structural stability of Anosov flows 436
structural stability of hyperbolic toral automorphisms 307
structural stability of Morse-Smale diffeomorphisms 354
Structural Stability Theorem 441
structurally stable 9, 45, 264
subadditive 63
subshift 75
subshift of finite type 75, 273, 274, 280
sup-norm 96, 143, 160
support 455
surface 262
suspension of a map 173
symbol space 38
symbolic dynamics 4, 38, 273, 279, 315
symplectic diffeomorphism 222
symplectic form 216
symplectic matrix 220
- tangent bundle 264
tangent space 264
tangent vector 264
Taylor's expansion 136
template 351
tent map 43, 58, 59, 61
Thom Transversality Theorem 452
topological conjugacy 39, 41

topological dimension 326
 topological entropy 87, 368
 topological Markov chain 75
 topologically conjugate flows 115
 topologically equivalent flows 115
 topologically mixing 40, 292
 topologically transitive 6, 40, 271
 topologically transverse 295
 toral Anosov diffeomorphism 306
 torus 262
 totally disconnected 26
 transition graph 68
 transition matrix 75, 274, 315
 transitive 40, 271
 transversal to a flow 168
 transversality condition 441
 Transversality Theorem, Parametric 452
 Transversality Theorem, Thom 452
 transverse 203, 352, 451
 transverse homoclinic point 285
 trapping region 324, 402
 tubular neighborhood 481
 two-shift 38
 two-sided shift space 273
 uniform hyperbolic structure 267

uniqueness of solutions of differential equations 143
 unstable cones 189
 unstable disk 188
 unstable eigenspace 113, 122
 unstable focus 107
 unstable manifold 185
 unstable manifold of an invariant set 415
 unstable node 107
 unstable set 16
 unstable spiral 107
 unstable subbundle 267
 unstable subspaces 184
 Van der Pol equation 5, 174
 variation of parameters 117
 vector bundle 467
 vector field 142, 266
 vertex subshift 274
 Volterra-Lotka equations 180
 weak Liapunov function 157, 357
 weak trapping region 403
 zeta function 80, 320