GENERALIZED ABRAHAM TRANSVERSALITY

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We use some of the notation from Section 11.2 on transversality. Assume that $\mathscr{A}$ is a topological space and manifolds M and N. The map $\rho : \mathscr{A} \to C^r(M, N)$ is called a C^r *pseudo-representation* provided that both ρ and $\rho^{\mathrm{ev}} : \mathscr{A} \times M \to N$ that is defined by $\rho^{\mathrm{ev}}(f, \mathbf{x}) = \rho(f)(\mathbf{x})$ are continuous.

Theorem 1. *Let M and N be finite dimensional second countable manifolds, K a compact subset of M a compact subset, and $V \subset N$ a closed C^1 submanifold of N. Assume that $\rho : \mathscr{A} \to C^r(M, N)$ is a C^1 pseudo-representation with $r \geq 1$, and let*

$$\mathscr{R} = \{\, f \in \mathscr{A} : \rho(f) \ \text{ is transverse to } \ V \,\}.$$

- **a.** *Then the subset $\mathscr{R}$ is an open subset of $\mathscr{A}$.*
- **b.** *If $\rho : \mathscr{A} \to C^r(M, N)$ is a C^r pseudo-representation with $r \geq \max\{\, 1, \ 1 + \dim(M) - \operatorname{codim}(V) \,\}$ and ρ^{ev} is transverse to V, then $\mathscr{R}$ is residual in $\mathscr{A}$ and hence dense.*

The proof of part (**a**) is basically the same as the openness in [1].

The most important assumption for part (**b**) is that ρ^{ev} is transverse to V. This assumption means that if $\rho(f)(\mathbf{x}) \in V$, then the space $\mathscr{A}$ of functions is large enough to be able to perturb f to be transverse to V at $\mathbf{x}$. The theorem then globalizes this result to give a perturbation that is transverse to V at all points of K. M. Hirsch communicated this version of the Abraham transversality theorem in a course in 1967. Unfortunately, his book [2] does not contain this general version. The proof uses the form of the Parametric transversality Theorem XI.2.3, and the compactness of K. The details are given in [3].

References

[1] R. Abraham and J. Robbin (1967), *Transversal Mappings and Flows*, Benjamin, New York.

[2] M. Hirsch (1976), *Differential Topology*, Springer-Verlag, New York.

[3] C. Robinson (1970), A global approximation theorem for Hamiltonian systems, in *Proc. Symposia in Pure Math. of the Amer. Math. Soc.*, (eds. S. S. Chern and S. Smale), Amer. Math. Soc., **14**, pp. 233–243.