115b/1 - Homework 1*

Due 10 January 2011

1. Let $P_4(\mathbb{R})$ be the vector space of degree 4 polynomials with real coefficients, and let $\beta = \{1, x, x^2, x^3, x^4\}$ be the standard ordered basis of $P_4(\mathbb{R})$. Let

$$T : P_4(\mathbb{R}) \to \mathbb{R}^1$$

be the linear transformation defined by

$$T(f(x)) = \int_0^{10} f(x)dx.$$

Compute $[T]_\beta^e$, the matrix representation of T with respect to β and e, where e is the standard basis $\{1\}$ of $\mathbb{R}^1$.

2. For all $n > 0$, construct an example of a nilpotent $n \times n$ matrix A such that $A^{n-1} \neq 0$, but $A^n = 0$.

3. Let A be an $n \times n$ nilpotent complex matrix. Prove that $A^n = 0$. (Hint: use Schur's theorem.)

4. Prove that if $T : V \to W$ is a linear transformation and V is finite dimensional, then

$$\dim(V) = rank(T) + nullity(T).$$

5. Prove that every $n \times n$ matrix A can be written

$$A = QLU,$$

where Q is a permutation matrix (a matrix having exactly one 1 in each row and column and zeros elsewhere), L is a lower triangular matrix, and U is an upper triangular matrix. (Hint: consider Gaussian elimination. Also, induction is always nice.)

6. Do problem (2.6.7).

7. Do problem (2.6.8).

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*Numbers in parentheses like (1.2.11) refer to the 11th problem in the second section of the first chapter of Friedberg *et. al.*