115b/1 - Homework 2*

Due 19 January 2011

1. Do problem (2.2.11).

2. Do problem (4.3.21).

3. Prove that if T is linear operator, then every eigenspace E_λ is T-invariant.

4. Let S and T be linear operators on the finite dimensional vector space V. Say that S and T are simultaneously diagonalizable if there is a basis β for V such that both $[S]_\beta$ and $[T]_\beta$ are diagonal matrices. Say that S and T commute if $S \circ T = T \circ S$. Show that if S and T do not commute, then S and T are not simultaneously diagonalizable.

5. In the situation of problem 4, show that if S and T are diagonalizable linear operators that commute, then they are simultaneously diagonalizable.

6. Do problem (5.4.13).

7. Do problem (5.2.2.d). Assume that the base field is $\mathbb{C}$.

*Numbers in parentheses like (1.2.11) refer to the 11th problem in the second section of the first chapter of Friedberg *et. al.*