33 AH - Practice Midterm 1

25 January 2012

1.a. (10 points) Let

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 3 \\ 9 \\ 7 \end{pmatrix}, \mathbf{v_2} = \begin{pmatrix} 2 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \mathbf{v_3} = \begin{pmatrix} 3 \\ -2 \\ 1 \\ 2 \end{pmatrix}, \mathbf{v_4} = \begin{pmatrix} -5 \\ 14 \\ 16 \\ 9 \end{pmatrix}.$$

Do the linear combinations of the vectors $\mathbf{v_1}, \ldots, \mathbf{v_4}$ span all of $\mathbb{R}^4$?

1.b (10 points) Show that any 3 of the vectors span a 3-plane in $\mathbb{R}^4$.

2.a. (5 points) Suppose that $A\mathbf{x} = \mathbf{b}$ has no solution (in particular $\mathbf{b} \neq \mathbf{0}$). Show that there are infinitely many $\mathbf{c}$ such that $A\mathbf{x} = \mathbf{c}$ has no solution.

2.b. (10 points) Suppose that A is an $n \times n$ matrix, but that elimination only produces $n-1$ non-zero pivots. Describe the space of $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$ has a solution.

2.c. (5 points) In the situation of part (b), why does $A\mathbf{x} = \mathbf{b}$ have infinitely many solutions if it has any?

3.a (10 points) Find a $PA = LU$ decomposition of

$$A = \begin{pmatrix} 0 & -1 & 2 \\ 2 & 0 & 3 \\ 4 & 1 & 6 \end{pmatrix}.$$

3.b (10 points) Write down a matrix for each row operation needed to find U from PA.

4. (20 points) Use Gauss-Jordan elimination to find A^{-1}, where

$$A = \begin{pmatrix} 1 & a & 0 \\ a & 1 & b \\ 0 & b & 1 \end{pmatrix}.$$

What conditions, if any, must a and b satisfy for A^{-1} to exist?