33 AH - Practice Midterm 2

17 February 2012

1.a. (10 points) Let

$$A = \begin{pmatrix} 0 & -1 & 1 & -1 & 3 \\ 4 & 0 & 3 & -1 & 1 \\ 4 & 2 & 6 & 1 & 7 \\ 2 & 1 & 2 & 1 & 6 \end{pmatrix}.$$

Find bases for the 4 fundamental subspaces associated to A.

1.b (10 points) Describe the complete solution to $A\mathbf{x} = \mathbf{b}$ where

$$\mathbf{b} = \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \end{pmatrix}.$$

2.a. (10 points) Write down a matrix A that has nullspace spanned by the two column vectors $\mathbf{v}_1 = (1, 2, 3, 4)$ and $\mathbf{v}_2 = (5, 6, 1, 3)$.

2.b. (10 points) Note: your matrix A from part (a) doesn't have independent columns. But, write down a projection matrix P that projects onto the column space of A.

3.a (10 points) Find the projection matrix P that projects onto the column space of

$$A = \begin{pmatrix} 0 & -1 & 2 \\ 2 & 0 & 3 \\ 4 & 1 & 6 \\ 1 & 1 & 1 \end{pmatrix}.$$

3.b (10 points) Give an example to show that two matrices with the same column spaces can have different projection matrices.

4. (20 points) Let $W \subseteq \mathbb{R}^n$ be a subspace and let $\mathbf{x}$ be a vector in $\mathbb{R}^n$. Show that if $\mathbf{p} \in W$ is a vector such that $\mathbf{p} - \mathbf{x}$ is orthogonal to W, then for every $\mathbf{y} \in W$, we have $||\mathbf{x} - \mathbf{p}|| \leq ||\mathbf{x} - \mathbf{y}||$. Then, show the converse as well. We have used these facts implicitely in class, but we haven't proven them.