Homework 7 - Math 132/3

Due 1 June 2012

1. Let $f_1 = 1$ and $f_2 = 1$, and define $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$. These are the Fibonnaci numbers. Determine the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{z^n}{f_n}.$$

Your answer should either be in terms of some well-known constants, or a decimal number computed to at least 4 significant places. You might consult the Wikipedia article entitled *Fibonacci number* for some potentially useful facts.

2. Can there be a non-constant function $f(z)$ analytic on the punctured unit disk $\{z|0 < |z| < 1\}$ such that $f(1/n) = 0$ for all integers $n > 1$? If not, prove it. If so, give an example of such a function and discuss whether the function is unique.

3. Compute the first 3 non-zero terms in the power series expansion of

$$f(z) = \frac{e^z}{(z-1)^4}$$

at $z = 2$.

4. Find all of the zeros of $f(z) = \sinh^2 z + \cosh^2 z$ and their orders. Compute the first 4 non-zero terms in the power series expansion of $f(z)$ at $z = 0$.

5. Show that any linear fractional transformation $f(z) = \frac{az+b}{cz+d}$ with $c \neq 0$ is analytic at ∞, and compute its power series expansion at ∞. There's a slick way to determine the interval of convergence: what is it?