Homework 8 - Math 132/3

Due 8 June 2012

1. Find a Laurent decomposition $f_0(z) + f_1(z)$ of the function

$$f(z) = \frac{1}{(z-1)^2(z-4i)}$$

on the annulus $\{z \mid 2 < |z| < 4\}$.

2. What is the radius of convergence for the Laurent series of $\frac{z}{\sin^3 z}$ at $z = \pi i$? Find the first 5 non-zero terms in the series.

3. Classify the singularities, including the determination of the orders of any pole and including what happens at ∞, of the functions

$$\frac{\operatorname{Log} z}{(z-1)^4} \qquad e^{\frac{z}{z^2+1}} \qquad z^3 \sin\left(\frac{1}{z^2-1}\right).$$

4. Evaluate the following integrals using the residue theorem:

$$\int_{|z|=4} \frac{z}{\sin z}\, \mathrm{d}z \qquad \int_{|z-1/2|=3/2} \frac{\tan z}{z^2}\, \mathrm{d}z$$

5. Show, using residue theory as in Gamelin section VII.2, that

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{(x^2+a^2)^2} = \frac{\pi}{2a^3}.$$