Problem set 6 for 131 A/3 - Fall 2012

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1. [Ros80, Exercise 28.2].
2. [Ros80, Exercise 28.3.a].
3. [Ros80, Exercise 28.8]. Let $f(x) = x^2$ when x is rational and $f(x) = 0$ when x is irrational. Show that f is continuous and differentiable at $x = 0$, but not continuous (and, hence, not differentiable) anywhere else.
4. [Ros80, Exercise 28.15].
5. Show that if f is differentiable on $[a, b]$ and if $f'(x) > 0$ on $[a, b]$, then f is strictly increasing.
6. Show that if f is differentiable at x then f is continuous at x.
7. Show that $|\cos x - \cos y| \leq |x - y|$ for all $x, y \in \mathbb{R}$.
8. Let $[a, b]$ be an interval, and $c \in (a, b)$. Suppose that f and g are two continuous functions on $[a, b]$ such that f is differentiable at c and g is not. Consider the question of whether or not the product fg is differentiable at c. This is an open-ended problem. You might need to find counterexamples or prove something.

References

[KF75] A. N. Kolmogorov and S. V. Fomīn, *Introductory real analysis*, Dover Publications Inc., New York, 1975. Translated from the second Russian edition and edited by Richard A. Silverman; Corrected reprinting.

[Nat55] I. P. Natanson, *Theory of functions of a real variable*, Frederick Ungar Publishing Co., New York, 1955. Translated by Leo F. Boron with the collaboration of Edwin Hewitt.

[Ros80] K. A. Ross, *Elementary analysis: the theory of calculus*, Springer-Verlag, New York, 1980. Undergraduate Texts in Mathematics.

[Rud87] W. Rudin, *Real and complex analysis*, 3rd ed., McGraw-Hill Book Co., New York, 1987.