Problem set 7 for 131 A/3 - Fall 2012

Benjamin Antieau

Due 30 November 2012

1. Let $f : [0, 1] \to \mathbb{R}$ be the function
$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational,} \\ \frac{1}{q} & \text{if } x = \frac{p}{q} \text{ and } p \text{ and } q \text{ are relatively prime.} \end{cases}$$
Prove that either f is or is not Darboux integrable.

2. Let $f : [0, 1] \to \mathbb{R}$ be the function
$$f(x) = \begin{cases} n & \text{if } x = \frac{1}{n}, \\ 0 & \text{otherwise.} \end{cases}$$
Prove that either f is or is not Darboux integrable.

3. Suppose that (f_n) is a sequence of bounded functions on an interval $[a, b]$ that converges pointwise to a function f on $[a, b]$. Determine with proof whether not the Darboux integrability of each f_n implies that f is Darboux integrable.

4. [Ros80, Exercise 32.7].

5. [Ros80, Exercise 32.8].

6. Prove [Ros80, Theorems 33.1 and 33.2] in your own words (it can be the "same" proof though).

7. [Ros80, Exercise 33.7].

8. [Ros80, Exercise 34.11].

9. Define what it should mean for a function $f : [a, \infty) \to \mathbb{R}$ to be Darboux integrable.

References

[KF75] A. N. Kolmogorov and S. V. Fomīn, *Introductory real analysis*, Dover Publications Inc., New York, 1975. Translated from the second Russian edition and edited by Richard A. Silverman; Corrected reprinting.

[Nat55] I. P. Natanson, *Theory of functions of a real variable*, Frederick Ungar Publishing Co., New York, 1955. Translated by Leo F. Boron with the collaboration of Edwin Hewitt.

[Ros80] K. A. Ross, *Elementary analysis: the theory of calculus*, Springer-Verlag, New York, 1980. Undergraduate Texts in Mathematics.

[Rud87] W. Rudin, *Real and complex analysis*, 3rd ed., McGraw-Hill Book Co., New York, 1987.