# 308F - Quiz 1

11 April 2014

Name:
Student ID #:

This is a closed-book, closed-notes exam. Calculators are not allowed.

**1. (1 point).** Is the matrix

$$B = \begin{pmatrix} 1 & 0 & 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

in Reduced Echelon Form?

**No, it must be in echelon form to be in reduced echelon form.**

**2. (1 point).** Is the matrix $B$ in Echelon Form?

**No, the trivial rows are not consolidated at the bottom. This is the only issue.**

**3. (2 points).** If the matrix is the augmented matrix of a system of linear equations, how many equations and how many unknowns are there? The matrix $B$ has size $x \times y$ for what $x$ and $y$?

**There are $x$ equations and $y - 1$ unknowns. Looking at the size of the matrix, it is a 6 x 6 matrix (6 rows and 6 columns) thus $x = y = 6$ and we have 6 equations and 5 unknowns. Note that trivial equations like: $0x_1 + 0x_2 + \ldots + 0x_n = 0$ are still equations. Also note that sizes of matrices are often indexed by the variables $m$ x $n$**

**4. (1 point).** Does the matrix $B$ represent a consistent system?

**No, we can see that row 5 has the form $0 = 1$ Which cannot be satisfied, thus the system is inconsistent. (See p. 19)**

**5. (5 points).** Find the Reduced Echelon Form of the augmented matrix corresponding to the system of linear equations

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_1 - x_2 &= 3 \\ 2x_1 + x_2 &= 2. \end{aligned}$$

Show all work.

**This system corresponds to the augmented matrix:**

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 2 & 1 & 2 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 2 & 1 & 2 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 2 & 1 & 2 \end{bmatrix} \xrightarrow{R_3 - 2R_1} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 0 & -1 & 0 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & -2 & 2 \end{bmatrix} \xrightarrow{(-1)*R_2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & -2 & 2 \end{bmatrix} \xrightarrow{R_3 + 2R_2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow{R_3/2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Note that this is not yet in R.E.F. the leading 1's are not the only non-zero entries in the columns.**

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 - 2R_2 - R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$