548.
Problem Set 1.
Due 21 January 2015.

1. Prove that $X \wedge S^0$ and X are homeomorphic as pointed spaces.

2. Prove that $S^1 \wedge X$ and ΣX (based suspension) are homeomorphic as pointed spaces.

3. Let (X, x) be a based space. Let X_+ denote $X \amalg \{+\}$, pointed by $+$. Show that there is a [redacted] homotopy equivalence

$$S^1 \wedge X_+ \simeq S^1 \wedge (X,x) \vee S^1.$$

4. Prove that $\pi_0(X, x)$ is indeed the set of connected components of X.

5. Prove that $\pi_1(S^1) \cong \mathbb{Z}$.

6. Prove that $\pi_n(S^1) = 0$ for $n > 1$. Hint: use the covering space $\mathbb{R} \to S^1$ and some kind of lifting property, as in Hatcher Proposition 1.33.

7. Show that if $f : S^{2n-1} \to S^n$ is nullhomotopic, then $C_f \simeq S^n \vee S^{2n}$, and hence that $H(f) = 0$. Here Cf denotes the mapping cone.