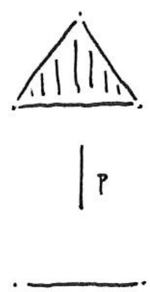
548.

Problem Set 3.

Due 11 February 2015.

1. Prove that $[(D^n, S^{n-1}, s), (X, A, x)]_* \cong \pi_{n-1}(P_x f)$, where $f : A \hookrightarrow X$. That is, show that the two definitions of $\pi_n(X, A, x)$ agree.

2. (Hatcher, Exercise 4.2.9.) Show that projection $\Delta^2 \xrightarrow{p} I^1$ is a fibration that is not a fiber bundle. One can realize



as $p(x_0, x_1, x_2) = (x_0, x_1)$, where $\Delta^2 \subseteq \mathbb{R}^3$ consists of (x_0, x_1, x_2) where $x_i \geq 0$ and $x_0 + x_1 + x_2 = 1$.

3. (Hatcher, Exercise 4.1.6.)

4. Let (Y, y) be a based space, and $f : \{y\} \to Y$ the inclusion of the basepoint. What is the fiber of f? What is the homotopy fiber of f?

5. Consider a commutative diagram
$$\begin{tikzcd} X \arrow[r, "f"] \arrow[d, "u", "\simeq"'] & Y \arrow[d, "v", "\simeq"'] \\ W \arrow[r, "g"] & Z \end{tikzcd}$$
in which the vertical maps are homotopy equivalences. Prove that hofib(f) is homotopy equivalent to hofib(g).

Assume also that
$$\begin{tikzcd} X \arrow[r] & Y \\ W \arrow[r] \arrow[u, "r"] & Z \arrow[u, "s"'] \end{tikzcd}$$
commutes, where r is the homotopy inverse to u, and s to v.

6. (Hatcher, Exercise 4.3.15.)

7. (Hatcher, Exercise 4.3.19.)

8. Prove that a pushout of a cofibration is a cofibration.