1. (Hatcher 4.3.1).

2. (Hatcher 4.3.5).

3. A vector bundle $E \to X$ is orientable if one can choose local trivializations $\phi_i : E_i \cong U_i \times \mathbb{R}^n$ such that the induced maps $\sigma_{ij} : U_{ij} \to GL_n(\mathbb{R})$ all have the same sign (± 1).

   Show that if M is an orientable manifold, as you learned for Poincaré Duality, the TM is orientable.

4. Compute the dimensions of the Stiefel varieties, [redacted] the Grassmannian varieties, and the orthogonal and unitary groups.

5. $PU_n = U_n / U_1$, where $U_1 \subseteq U_n$ is the subgroup consisting of matrices $\begin{pmatrix} \xi & & 0 \\ & \ddots & \\ 0 & & \xi \end{pmatrix}$, $|\xi| = 1$. Show that $PU_3 \cong SO_3$ as topological groups. Recall that $SO_3 \subset O_3$ is the subgroup of orthogonal matrices of determinant 1.