MATH 215 – Number Theory II (NTII)

You will want to refer back to NTI when working through these problems.

Definition 1 *Two integers a and b are called* **relatively prime (or coprime)** *if* $\gcd(a, b) = 1$.

Proposition 2 *Two integers a and b are relatively prime if and only if there exists integers m, n such that $am + bn = 1$.*

Proposition 3 *Let a, b be relatively prime integers.*

- *If $a|c$ and $b|c$, then $ab|c$.*
- *If $a|bc$, then $a|c$.*

Definition 4 *An integer $p > 1$ is* **prime** *if the only positive divisors of p are 1 and p.*

Proposition 5 *Let p be a prime, and let a be an integer. Then either $p|a$ or p and a are relatively prime.*

Proposition 6 *Let p be a prime, and let a, b be integers. If $p|ab$, then $p|a$ or $p|b$.*

Corollary 7 *If p is prime, and $p|a_1 \cdot a_2 \cdots a_{k-1} \cdot a_k$, then $p|a_i$ for some $i = 1, 2, \ldots, k$.*

Theorem 8 (Fundamental Theorem of Arithmetic) *Let n be an integer greater than 1. Then*

$$n = p_1^{e_1} \cdots p_k^{e_k}$$

where the primes $p_1 < p_2 < \cdots < p_k$ are distinct and the exponents e_i are positive integers. This **prime-factorization** *is unique.*

Theorem 9 (Euclid's Theorem) *There are infinitely many primes.*

Hint: There are MANY proofs that there are an infinite number of primes, but Euclid's proof is the most beautiful (it is short and sweet). It is perhaps the "prettiest" proof in all of mathematics. To arrive at a contradiction, assume that there are only finitely many primes. Call those primes $p_1, p_2, \ldots, p_k$, and consider the integer $n = p_1 p_2 \cdots p_k + 1$.