547 - Spring 2018 - HW3

February 5, 2018

1. Prove that if $E \xrightarrow{p} B$ and $F \xrightarrow{q} B$ are covering spaces, then so is $E \coprod F \xrightarrow{p \coprod q} B$.

2. Show that if $E \xrightarrow{p} B$ is a covering space and $X \to B$ is a map with X connected and locally path connected, then $E \times_B X \to X$ is a covering space.

3. Let G be a discrete group acting freely and continuously on a topological space X. Assume that X is locally path connected. Prove that $X \to X/G$ is a covering space. What are the fibers?

4. Suppose that G is a discrete group acting freely on a simply connected space X. Compute $\pi_1(X/G)$.

5. Suppose that G and H are non-zero groups. Find the center of $G * H$.

6. Let X be the complement of finitely many points in $\mathbb{R}^n$ for $n \geqslant 3$. Compute $\pi_1(X)$.

7. Let X be the complement of finitely many points in $\mathbb{R}^2$. Compute $\pi_1(X)$.

8. Let X be a path-connected space and consider a map $S^n \xrightarrow{f} X$ where $n \geqslant 2$. Compute $\pi_1(C_f)$.

9. Prove that $\pi_1(\mathbb{R}^2 - \mathbb{Q}^2)$ is uncountable.

10. Do Hatcher, Exercise 1.2.7.