547 - Spring 2018 - Outline

May 14, 2018

Categories.

1. Categories.
2. The category $\mathcal{T}$ of topological spaces.
3. Comma categories.
4. The category $\mathcal{T}_*$ of pointed topological spaces.
5. Functors.
6. Pushouts and pullbacks.

Homotopy.

1. Homotopy equivalence.
2. The homotopy categories $\mathrm{Ho}(\mathcal{T})$ and $\mathrm{Ho}(\mathcal{T}_*)$ of topological spaces (resp. pointed topological spaces).
3. The fact that $\mathrm{Ho}(\mathcal{T})$ is actually a category.
4. Deformation retracts.
5. Contractibility of the infinite-dimensional sphere S^∞.
6. Examples of spaces that do not retract onto subspaces.

Fundamental groups.

1. Homotopy of paths.
2. The fundamental groups.
3. Independence of base point.
4. The fundamental group as a functor $\mathcal{T}_* \to$ **Groups**.
5. The fundamental group as a functor $\mathrm{Ho}(\mathcal{T}_*) \to$ **Groups**.
6. $\pi_1 S^1 \cong \mathbb{Z}$.

Covering spaces.

1. Definition.
2. Uniqueness of lifting paths.
3. Categories of (pointed) covering spaces over a base space.
4. Existence of the universal cover.
5. The Galois theory for covering spaces.
6. Classification of regular G-covers (as in Fulton).
7. The correspondence between regular G-covers and group homomorphisms $\pi_1(X) \to G$.
8. $\pi_1(X/G)$ for X simply connected and G acting freely and discontinuously on X.
9. $\pi_1\mathbb{RP}^n$ for $n \geqslant 2$.

Van Kampen.

1. Coproducts and free products of groups.
2. Van Kampen's theorem via the universal property above.
3. Applications of Van Kampen's theorem.
4. $\pi_1\mathbb{CP}^2 = 0$.
5. $\pi_1 S^n = 0$ for $n > 1$.
6. $\pi_n S^1 = 0$ for $n > 1$.
7. π_1 of the orientable Riemann surface of genus g.

Homological algebra.

1. Chain complexes.
2. Homology.
3. Snake lemma. 3x3 lemma. 5-lemma.
4. Injective and projective objects.
5. Injective and projective resolutions.
6. Comparison of resolutions.
7. Ext and Tor.
8. $\mathrm{Tor}_*^{k[\epsilon]/(\epsilon^2)}(k,k)$.
9. $\mathrm{Tor}_1^{\mathbb{Z}}(M,N)$ for abelian groups M,N. Same for $\mathrm{Ext}^1_{\mathbb{Z}}(M,N)$.

Homology.

1. Simplicial sets.
2. The singular simplicial set.
3. Simplicial abelian groups and their associated chain complexes.
4. Chains on a space, possibly with coefficients.
5. Relative homology.
6. CW homology.
7. Excision.
8. Reduced homology $\widetilde{\mathrm{H}}_n$.
9. If $A \subseteq X$ is a pair where A has an open neighborhood of which it is a deformation retract, then $\mathrm{H}_*(X, A) \cong \widetilde{\mathrm{H}}_*(X/A)$.
10. Universal coefficients.

Cohomology.

1. Universal coefficients.
2. Cup product.
3. Graded commutativity.
4. The cohomology ring of closed orientable Riemannian surfaces.

Poincaré duality.

1. R-orientability.
2. The orientation covering space.
3. The top homology groups of closed n-manifolds.
4. Cap product.
5. Cohomology with compact supports.
6. Poincaré duality.
7. The cohomology pairing.
8. Deduction of cohomology ring structures for spaces like $\mathbb{CP}^n$ from Poincaré duality.