548 - Spring 2018 - HW2

January 26, 2018

1. Let $\mathcal{C}$ be a category with finite products, meaning that for any finite set $\{X_i\}_{i\in I}$ of objects X_i of $\mathcal{C}$, the product $\prod_{i\in I} X_i$ exists. Prove that $\mathcal{C}$ has a final object.

2. Prove that ΣX is an H-cogroup in $\mathcal{T}_*$ for any $X \in \mathcal{T}_*$.

3. Prove that ΩX is an H-group for any $X \in \mathcal{T}_*$.

4. Prove that if G is a group object in a category $\mathcal{C}$, then $\mathrm{Hom}_{\mathcal{C}}(X, G)$ is naturally a group for every X in $\mathcal{C}$.

5. Prove that if C is a cogroup object in a category $\mathcal{C}$, then $\mathrm{Hom}_{\mathcal{C}}(C, X)$ is naturally a group for every X in $\mathcal{C}$.

6. Let Δ^1 denote the category with two objects 0 and 1 and a unique non-identity morphism $f : 0 \to 1$. Let $\mathcal{C}$ be another category. Describe the functor category $\mathrm{Fun}(\Delta^1, \mathcal{C})$.

7. Using the universal property discussed in class, identify $\mathrm{Fun}(\Delta^1[W^{-1}], \mathcal{C})$ where $W = \{f\}$.

8. Prove that the localization $\Delta^1 \to \Delta^1[W^{-1}]$ exists by exhibiting an explicit category $\mathcal{C}$ with a functor $\Delta^1 \to \mathcal{C}$ which has the correct universal property.