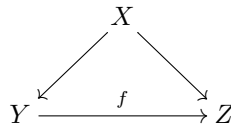


# 548 - Spring 2018 - HW3

February 2, 2018

1. Prove that in  $\text{Ch}_{\geq 0}(A)$  every morphism  $X \xrightarrow{f} Z$  factors as  $X \xrightarrow{i} Y \xrightarrow{p} Z$  where  $i \in C \cap W$  and  $p \in F$ . (Every morphism factors as an acyclic cofibration followed by a fibration.) This completes the proof that  $\text{Ch}_{\geq 0}$  is a model category.
2. Let  $\mathcal{C}$  be a model category. Prove that cofibrations are closed under cobase change. In other words, prove that if  $X \xrightarrow{i} Y$  is a cofibration and  $X \xrightarrow{f} Z$  is any map, then the pushout  $Z \xrightarrow{j} Z \cup_X Y$  is a cofibration.
3. Prove that if  $\mathcal{C}$  is a model category (with respect to some classes  $W, C, F$ ), then  $\mathcal{C}^{\text{op}}$  admits a natural model category structure.
4. Suppose that  $\mathcal{C}$  is a model category. Find a model category structure on  $\mathcal{C}$  where the class  $W$  of weak equivalences is the class of isomorphisms in  $\mathcal{C}$ .
5. Based on the work in class with  $\text{Ch}_{\geq 0}(A)$ , propose a model category structure on  $\text{Ch}^{\geq 0}$ , the category of non-negatively graded *cochain* complexes of left  $A$ -modules.
6. Prove that if  $\mathcal{C}$  is a model category (with respect to  $W, C, F$ ), then for any object  $X$  of  $\mathcal{C}$ ,  $\mathcal{C}_{X/}$  admits a model category structure where a map



in  $\mathcal{C}_{X/}$  is a weak equivalence, cofibration, or fibration if and only if  $f$  is a weak equivalence, cofibration, or fibration in  $\mathcal{C}$ .

**Definition 0.1.** Let  $\mathcal{C}$  be a model category and let  $X \in \mathcal{C}$  be an object. A **cylinder object** for  $X$  is an object  $X \wedge I$  of  $\mathcal{C}$  (this is just a formal symbol) together with maps  $X \amalg X \rightarrow X \wedge I$  and  $X \wedge I \xrightarrow{\sim} X$  whose composition is the fold map  $X \amalg X \rightarrow X$  induced by  $X \xrightarrow{\text{id}_X} X$  and  $X \xrightarrow{\text{id}_X} X$ . A cylinder object is **good** if  $X \amalg X \rightarrow X \wedge I$  is a cofibration and **very good** if additionally  $X \wedge I \xrightarrow{\sim} X$  is a fibration, which is necessarily acyclic.

7. Prove that very good cylinder objects exist for every object  $X$  of a model category  $\mathcal{C}$ .