548 - Spring 2018 - HW4

February 9, 2018

1. Given a chain complex $X \in \mathrm{Ch}_{\geqslant 0}(A)$, construct a very good cylinder object for X.

2. Prove that if X is fibrant, $f \overset{\ell}{\sim} g \colon A \to X$ are two left homotopic morphisms, and $h : A' \to A$ is any morphism, then $f \circ h \overset{\ell}{\sim} g \circ h$.

3. Prove that if X is fibrant, the assignment $([f], [g]) \mapsto [g \circ f]$ is well-defined for $f : A' \to A$ and $g : A \to X$, giving a composition function

$$\pi^{\ell}(A', A) \times \pi^{\ell}(A, X) \to \pi^{\ell}(A', X).$$

4. Let $\mathcal{C}$ be a model category. Prove that $f : X \to Y$ maps to an isomorphism in $\mathrm{Ho}(\mathcal{C})$ if and only if $f \in W$.

5. Prove using only what we've done with model categories that if $f : M \to N$ is a morphism of two left A-modules and if $P_* \to M$ and $Q_* \to N$ are projective resolutions, then there exists a morphism $\tilde{f} : P_* \to Q_*$ making

$$\begin{array}{ccc} P_* & \xrightarrow{\tilde{f}} & Q_* \\ \downarrow & & \downarrow \\ M & \xrightarrow{f} & N \end{array}$$

commute.

6. Let A be an associative ring and let M and N be two left A-modules. View M and N as chain complexes concentrated in degree 0. Compute $\mathrm{Hom}_{\mathrm{Ho}(\mathrm{Ch}_{\geqslant 0}(A))}(M, N)$.

7. Let M and N be left A-modules. Denote, for $n \geqslant 0$, by $N[n]$ the chain complex with N in degree n and zeros elsewhere. Compute $\mathrm{Hom}_{\mathrm{Ch}_{\geqslant 0}(A)}(M, N[n])$.

8. In the situation of Problem 7, compute $\mathrm{Hom}_{\mathrm{Ho}(\mathrm{Ch}_{\geqslant 0}(A))}(M, N[n])$.