# 547 - Fall 2019 - HW7

Due Monday 4 November 2019

**1.** Prove the simplicial identities as shown at `https://ncatlab.org/nlab/show/simplicial+identities`.

**2.** Recall that if $M$ is a compact metric space and $\mathscr{U} = \{U_i\}_{i \in I}$ is a cover, then a **Lebesgue number** is an $\epsilon > 0$ such that if $D \subseteq M$ is a subset with diameter less than $\epsilon$, then $D \subseteq U_i$ for some $i$. Prove that Lebesgue numbers exist. (Note: this one is harder than usual.)

**3.** Compute the homology of $\mathbb{CP}^n$ with a single point removed.

**4.** Recall that $\mathbb{RP}^n$ is covered by $n+1$ standard open sets $U_i$, $0 \leqslant i \leqslant n$, each of which is isomorphic to $\mathbb{R}^n$. We have that $U_i$ consists of lines through the origin containing points $(x_0, \ldots, x_n)$ where $x_i \neq 0$. Compute the cohomology of the union of $U_0$ and $U_1$ inside $\mathbb{RP}^3$.

**4.** Hatcher, Exercise 2.2.2.

**5.** Hatcher, Exercise 2.2.3.

**6.** Hatcher, Exercise 2.2.4.

**7.** Hatcher, Exercise 2.2.12.

**8.** Hatcher, Exercise 2.2.23.