

Math 331 discussion problems

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These are extra practice problems, not to be handed in.

1. Prove or disprove that $(2, x, y)$ is a maximal ideal in $\mathbb{Z}[x, y]$.
2. Let R be the set of infinite sequences $r = (r_1, r_2, r_3, \dots)$ of real numbers with the property that there exists N (depending on r) such that $r_m = r_N$ for all $m \geq N$. Observe that R is a ring under componentwise addition and multiplication.
 - (a) Describe R^\times .
 - (b) Determine all maximal ideals of R .
3. Let

$$R = \left\{ r \in \mathbb{Q} \mid \exists a \in \mathbb{Z}, n \in \mathbb{Z}_{\geq 0} \text{ s.t. } r = \frac{a}{p^n} \right\}$$

Check that R is an integral domain and find its field of fractions.