MATH 465-2, WINTER 2014: FACTORIZATION ALGEBRAS

J. FRANCIS

This course will give an exposition of factorization algebras in both the topological and algebro-geometric settings, culminating in recent work of Gaitsgory & Lurie that uses factorization homology methods to prove Weil's conjecture on Tamagawa numbers for the case of function fields. The following is a provisional outline:

(1) **Topology**
- Disk algebras and the equivalence of $\mathsf{Disk}(M)$-algebras and factorizable cosheaves on the Ran space $\mathsf{Ran}(M)$ for a manifold M.
- Poincaré/Koszul duality and Verdier duality on the space $\mathsf{Ran}(M)$.
- Factorization homology of commutative algebras and enveloping algebras of Lie algebras.
- The Atiyah-Bott formula for the singular homology of the space of G-bundles on a topological surface à la factorization homology.

(2) **Algebraic Geometry**
- Factorization algebras in ℓ-adic sheaves on $\mathsf{Ran}(X)$ for an algebraic variety X. The affine Grassmannian.
- Verdier/Koszul duality on the stack $\mathsf{Ran}(X)$.
- The Atiyah-Bott formula for the ℓ-adic homology of the stack of G-bundles on an algebraic curve X à la factorization homology.
- Tamagawa measure and the trace of Frobenius on the cohomology of $\mathsf{Bun}_G(X)$ for a curve X over a finite field $\mathbb{F}_q$.

Prerequisites: Participants should have a knowledge of algebraic topology and algebraic geometry at the level of year-long introductory graduate courses (e.g., Hatcher and Hartshorne).

References

[1] Ayala, David; Francis, John. Poincaré/Koszul duality. Preprint in preparation.
[2] Behrend, Kai. The Lefschetz trace formula for the moduli stack of principal bundles. PhD dissertation.
[3] Beilinson, Alexander; Drinfeld, Vladimir. Chiral algebras. American Mathematical Society Colloquium Publications, 51. American Mathematical Society, Providence, RI, 2004.
[4] Francis, John; Gaitsgory, Dennis. Chiral Koszul duality. Selecta Math. (N.S.) 18 (2012), no. 1, 27–87.
[5] Gaitsgory, Dennis. Contractibility of the space of rational maps. Inventiones Math. 191 (2013), no. 1 , 91–196.
[6] Lurie, Jacob. Course notes for Math 283: Tamagawa numbers via nonabelian Poincaré Duality.
[7] Lurie, Jacob. Derived algebraic geometry VI: $\mathbb{E}[k]$-algebras. Preprint.