

A Brief Course in Linear Algebra

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Evanston, Illinois
2002

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Preface

This is a basic linear algebra text used for many years at Northwestern University in the fifth quarter of the calculus sequence. The subject matter is introductory linear algebra. An attempt is made to be brief with emphasis on important basic concepts.

Linear algebra is somewhat more theoretical than other subjects in typical calculus courses. Some of these concepts are subtle, and instead of memorizing formulas one must learn moderately complex procedures for solving problems. In developing the subject matter, I have tried to keep things concrete by concentrating on illustrative examples. Such examples exhibit the important features of the theory. To describe a concept in complete generality will often require an extensive discussion and listing of many special cases, but someone with a good understanding of many examples will see what to do in less familiar circumstances.

There aren't as many exercises as one might find in a typical calculus text. Most of the exercises take a bit longer to do, so one must glean as much as possible from each rather than relying on repetition to drive a point home. There are fairly complete answers at the end of the book. Linear algebra, by its very nature, tends to be more theoretical than other topics in the calculus, and some of the exercises reflect that. In most cases it is not necessary to write out a formal proof; a convincing explanation will suffice.

In order to be brief, I haven't included many of the beautiful and important applications of linear algebra. A few such applications are mentioned in the exercises, and two important applications are included at the end. However, linear algebra is one of the most essential mathematical tools in science, engineering, statistics, economics, etc., so it is important to master its concepts and techniques.

For completeness, and for those who are interested, we have included almost all proofs of crucial theorems, but they can be skipped on a first reading. Given that time is limited, it is not unreasonable to postpone the proofs for a more advanced course in linear algebra.

No one has ever written a perfect book. A publisher once told me that people still find typographical errors in the oft reprinted works of Charles Dickens. If you find something that doesn't seem to make any sense, in either the text or the problems, please bring it to my attention at len@math.northwestern.edu.

I would like to thank Professor Daniel S. Kahn who helped greatly with the preparation of this text but who doesn't want to be held responsible, as an author, for my misdeeds. I would also like to thank my teaching assistants for valuable comments.

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as any resulting documents also follow the GFDL. The GFDL is included as an appendix.

There are many commercially available linear algebra texts, some of which have something unique to contribute. I don't object to those who wish to profit from making a real contribution to teaching mathematics. But we certainly don't need any more clones of existing textbooks which compete with one other in a crowded marketplace. I hope at least one freely available text will make a contribution toward keeping that number down.

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