

Math 300: Foundations of Higher Mathematics

Northwestern University, Spring 2017

Course Information

- Instructor: Santiago Cañez
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- Website: <https://canvas.northwestern.edu/courses/21107/>
- Office Hours: TTh 1-3pm in Lunt B27, or by appointment
- Lecture: MWF 1-2pm in Lunt 107
- Textbook: *Book of Proof* by Richard Hammack
- Prerequisites: Math 240 or a similar linear algebra course

Topics Covered

Mathematical statements and logic, Proof techniques and induction, Set theory and functions, Equivalence relations and partitions, Cardinality

What Is This Course About?

This course focuses on the “foundational” concepts which bind all of mathematics together. These are all ideas with which you will come across again and again in your advanced math courses, and the point is that rather than having to relearn it all every single time we will master it all here.

Why would we want to do this? Consider the following question: What is mathematics? If you asked numerous people this question it is likely that you would get a wide range of responses. Yes, some responses might be more common than others, but you may very well be surprised as to how different some of these responses are from each other. Similarly, you might be surprised as to how different-seeming some higher level math courses actually are. How on earth can we even talk about mathematics then when it seems to lead to so many “interpretations”? The answer is that regardless of all these “interpretations”, they all share a common core and that is what we will attempt to zero in on. We will talk about the way in which mathematics is communicated, whether it be through notation, terminology, in writing, or through the all important “proof”, which is similar in spirit but practically quite different than proofs you may have done in a high school geometry class. We will also get a glimpse into the nature of modern mathematics and what it is exactly that mathematicians do. We will move away from the more computational types of questions you may be used to coming from earlier math courses and more towards conceptual questions which require good reasoning skills.

In order to wet your appetites, here is a brief description of the final topic we will cover: cardinality. Cardinality is all about understanding the size of a set. Clearly, this is a simple concept for sets with only finitely many things in them—you simply count the number many things you have. However, what about the size of an *infinite* set? Well, you might just say the answer is infinity, but the point is that this is an inadequate answer since, amazingly, different infinite sets can still have different “sizes”. In other words, there are different “types” of infinity when it comes to describing size! Understanding the mathematical ideas which go into giving a precise definition of things such as the “size” of an infinite set and its paradoxical consequences is part of what this course is all about.

One final note: in this class we will be doing much writing, perhaps more writing than you've done in any other math course. Indeed, those of you who thought you would never have to think about sentence and paragraph structure again after your English courses may be surprised to learn that mathematicians think about this stuff all the time. After all, what good is understanding a certain mathematical idea if you're not able to communicate that idea to others? If you were to ever look at a published mathematical research article you may be surprised to see how many actual writing there is and relatively few formulas or equations. We'll spend a good amount of time perfecting our mathematical grammar, and your instructors-to-be in advanced math courses will be very grateful as a result.

What Should You Already Know?

Not much. Technically the only prerequisite is Math 240 or a similar linear algebra course, but this is really just to make sure you have the right "mathematical maturity" for this course. There's really not much from previous courses we will be using, although some topics from calculus and algebra may make an appearance from time to time. More important is the motivation and drive to learn this stuff, which at times might seem kind of dry. I'll try my best to make it all worthwhile.

Homework, Quizzes, and Exams

Homework will (usually) be due on Fridays. You are welcome (and encouraged!) to work together on problem sets but each student must hand in their own assignment in their own writing. If you do work with others, please list the names of those with whom you worked on the front page of your homework.

Discussion will be devoted to working on additional problems together as a class and in groups. You will be required to turn in a solution to one of the problems introduced that day, which will be graded solely for attempt and not correctness. The point is that I want to see how your writing is progressing week-to-week without the added pressure of getting things perfectly correct; if you turn in a reasonable attempt, even if headed in the wrong direction, you will get credit.

There will be three exams: Midterm 1 on Tuesday, April 25 in discussion, Midterm 2: Tuesday, May 16 in discussion, and the Final on Monday, June 5, 9-11am.

Grades

Your final score will be composed of homework and exam scores according to the following percentages: 30% Homework, 20% Quizzes, 20% Midterm, 30% Final Exam. What constitutes an A, B, etc. will be determined at the end once all scores have been totaled, so there is no set scale. However, I'll try to give a sense of where you stand throughout the quarter.

University Policies

Students are required to abide by Northwestern University's academic integrity policy, which can be found at <http://www.northwestern.edu/provost/students/integrity/>. Failure to adhere to this policy will likely result in a failing grade in the class and/or expulsion from the University.

Any student requesting accommodations related to a disability or other condition is required to register with AccessibleNU (847-467-5530) and provide professors with an accommodation notification from AccessibleNU, preferably within the first two weeks of class. All information will remain confidential.