# Northwestern University

Math 224 Final Exam Fall Quarter 2017 December 6, 2017

Last name:	Email address:
First name:	NetID:

Instructions

- This examination consists of 17 pages, not including this cover page. Verify that your copy of this examination contains all 17 pages. If your examination is missing any pages, then obtain a new copy immediately.
- This examination consists of 10 questions for a total of 200 points.
- You have two hours to complete this examination.
- Do not use books, notes, calculators, computers, tablets or phones.
- Write legibly and only inside of the boxed region on each page.
- Cross out any work that you do not wish to have scored.
- Show all of your work. Unsupported answers may not earn credit.

- 1. Determine whether each of the following statements is **TRUE** or **FALSE**, and circle your choice. You do not need to justify your answers.
  - (a) (3 points) If the sum  $\sum_{n=0}^{\infty} c_n 5^n$  converges, then  $\sum_{n=0}^{\infty} (-1)^n c_n 2^n$  converges as well.

TRUE FALSE

(b) (3 points) If  $T_4(x)$  is the 4-th degree Taylor polynomial of a function f(x) centered at a = 1, then  $f(x) - T_4(x) = \frac{f^{(5)}(0)}{5!}(x-1)^5$ .

#### TRUE FALSE

(c) (3 points) If 
$$\int_{1/2}^{\sqrt{2}/2} f(x) \, dx = -2$$
 then  $\int_{\pi/6}^{\pi/4} f(\sin x) \cos x \, dx = -2$ .

TRUE FALSE

(d) (3 points) If  $a_n \ge 0$  for all n and the series  $\sum_{n=0}^{\infty} a_n$  converges, then  $\sum_{n=0}^{\infty} \frac{a_n}{n}$  also converges.

#### TRUE FALSE

(e) (3 points) If f(x) is a continuous function on [a, b], the left endpoint Riemann sum approximation for  $\int_{a}^{b} f(x) dx$  for a given partition is larger than the right endpoint Riemann sum approximation for that same partition.

TRUE FALSE

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2. (20 points) It is known that  $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ . Use this information to evaluate the following integral.

$$\int_0^\infty x^2 e^{-x^2} dx$$

**Hint:** It may help to rewrite the integrand as  $x(xe^{-x^2})$ .

3. (17 points) The following is the graph of a function g(x) on the interval [-4.4].



Consider the function f(x) on the interval  $[e^{-4}, e^4]$  defined by

$$f(x) = \int_0^{\ln x} g(t) dt.$$

Find all x in the interval  $[e^{-4}, e^4]$  satisfying f'(x) = 0.

4. (20 points) Evaluate the following integral.

$$\int \frac{1}{\sqrt{x^2 - 2x + 5}} \, dx$$

**Hint:** Find values of a, b satisfying  $x^2 - 2x + 5 = (x - a)^2 + b^2$ .

5. Let R be the region in the first quadrant of the xy-plane bounded by the curves  $y = x^3$  and  $y = \sqrt{x}$ . (a) (10 points) Find the area of R.

(b) (10 points) Set up an integral which computes the volume of the solid obtained by revolving R about the y-axis. You do **not** have to evaluate this integral.

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- 6. This problem has two parts; the second is on the next page.
  - (a) (12 points) Determine whether the series

$$\sum_{n=2}^{\infty} \frac{\sqrt{n+1}}{n^2 + n - 2}$$

converges or diverges. Be clear about which convergence test you are using.

(b) (12 points) Determine whether the improper integral

$$\int_{\frac{1}{2}}^{1} \frac{1}{x(x-1)} dx$$

converges or diverges. If it converges, find its value.

- 7. This problem has two parts; the second is on the next page.
  - (a) (12 points) Find the Maclaurin series of the function  $f(x) = x \ln(1+x^2)$  by somehow manipulating

the series 
$$\frac{1}{1-y} = \sum_{n=0}^{\infty} y^n$$

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(b) (10 points) Determine the value of the following infinite sum. You must also give a reason as to why the sum actually converges.

$$\sum_{n=0}^{\infty} \frac{(-1)^{3n+3}}{2^{2n+3}(n+1)}$$

Hint: Evaluate the series from part (a) at an appropriate value of x.

8. (a) (10 points) Suppose f(x) is a function satisfying  $f^{(n)}(-1) = \frac{(n+3)!}{3^n}$  for each  $n \ge 0$ . Find the Taylor series of f(x) centered at a = -1.

(b) (10 points) Find a single function f(x) satisfying all of the following conditions:

f(0) = 3, f'(0) = -11, f''(0) = 9, f'''(0) = 5,  $f^{(4)}(0) = -3$ ,  $f^{(9)}(0) = 22$ 

- 9. This problem has two parts; the second is on the next page.
  - (a) (10 points) Find the Maclaurin series of the following function.

 $f(x) = x(\cos x - 1)$ 

(b) (12 points) Find a bound on the error obtained when approximating f(x) using its third degree Taylor polynomial  $T_3(x)$  centered at 0 on the interval  $\left[-\frac{1}{10}, \frac{1}{10}\right]$ . Here, f(x) is the same function as in part (a).

10. (20 points) Determine the interval of convergence of the following power series.

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{4^n(n+2)} (x+1)^n$$

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If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score.

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