

Northwestern University

Math 224 Midterm 1

Fall Quarter 2017

October 16, 2017

Last name: _____ Email address: _____

First name: _____ NetID: _____

Instructions

- This examination consists of 11 pages, not including this cover page. Verify that your copy of this examination contains all 11 pages. If your examination is missing any pages, then obtain a new copy immediately.
- This examination consists of 6 questions for a total of 100 points.
- You have one hour to complete this examination.
- Do not use books, notes, calculators, computers, tablets or phones.
- Write legibly and only inside of the boxed region on each page.
- Cross out any work that you do not wish to have scored.
- Show all of your work. Unsupported answers may not earn credit.

The following identities may be helpful:

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$

$$\int \tan x \, dx = \ln |\sec x| + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \frac{1}{1+x^2} \, dx = \arctan x + C$$

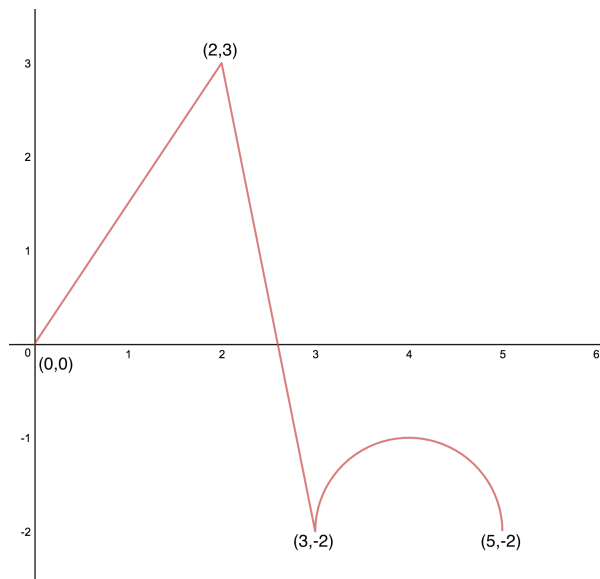
$$\sec^2 x = \tan^2 x + 1$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

1. This question has five parts; the rest are on the next page. Determine whether each of the following statements is **TRUE** or **FALSE**, and circle your choice. You do not need to justify your answers.

(a) (3 points) Consider the following graph of a function $g(t)$.



The function $G(x) = \int_0^x g(t) dt$ has a maximum at $x = 2$.

TRUE

FALSE

- (b) (3 points) Let f be a decreasing function on an interval $[a, b]$. If R is the value of a Riemann sum for f defined using right endpoints as sample points, then $R \leq \int_a^b f(x) dx$.

TRUE

FALSE

- (c) (3 points) If f and g are continuous functions on an interval $[a, b]$, then the identity

$$\int_a^b f(x)g(x) dx = \left(\int_a^b f(x) dx \right) \left(\int_a^b g(x) dx \right)$$

always holds.

TRUE

FALSE

- (d) (3 points) If $\int_1^e f(x) dx = 4$, then $\int_0^1 e^x f(e^x) dx = 4$.

TRUE

FALSE

- (e) (3 points) If f is continuous and $F(x) = \int_1^{x^2} f(t) dt$, then $F'(x) = f(x^2)$.

TRUE

FALSE

2. (a) (8 points) Find a function $f(x)$ and an interval $[a, b]$ such that

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left(\cos^2 \left(2 + \frac{i}{n} \right) + \left(2 + \frac{i}{n} \right) \right)$$

equals the value of $\int_a^b f(x) dx$.

- (b) (8 points) Find the value of this limit by evaluating the integral found in part (a). You do not have to simplify your answer.

3. This question has two parts; the second is on the next page. Evaluate the following indefinite integrals.

(a) (12 points) $\int \frac{e^{2x}}{e^{4x} + e^{2x}} dx$

(b) (12 points) $\int \frac{1}{x^2 \sqrt{4 - x^2}} dx$

4. An oven thermometer has the following readings over a 30 minute period

t	0	5	10	15	20	25	30
$T(t)$	347	350	361	365	370	369	372

where $T(t)$ is the temperature at time t . Estimate the average temperature over this time in the following two ways:

- (a) (8 points) Use the midpoint rule, for $n = 3$ intervals. You do not have to simplify your answer.

- (b) (8 points) Use Simpson's rule, for $n = 6$ intervals. You do not have to simplify your answer.

5. (15 points) Suppose a function f is differentiable and satisfies $f(6) = \frac{1}{3}$, $f(2) = \frac{1}{2}$, and

$$\int_2^6 f(x) dx = -1.$$

Find the value of

$$\int_4^{36} f'(\sqrt{x}) dx.$$

Hint: Try the substitution $t = \sqrt{x}$.

6. (14 points) Evaluate the definite integral

$$\int_0^1 x \arctan x \, dx.$$

Hint: At some point, you may need to use polynomial long division.

YOU MUST SUBMIT THIS PAGE.

If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score.

YOU MUST SUBMIT THIS PAGE.

If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score.

DO NOT WRITE ON THIS PAGE.