

Northwestern University

Math 226 Final Exam
Fall Quarter 2019
December 13, 2019

Name:

Initials:

netID:

Instructions

- This examination consists of 10 pages, not including this cover page. Verify that your copy of this examination contains all 10 pages. If your examination is missing any pages, then obtain a new copy of the examination immediately.
- Enter your initials in the indicated box on each page, and enter your Name and netID on the indicated boxes on the cover sheet.
- This examination consists of 8 questions for a total of 100 points.
- You have 2 hours to complete this examination.
- Do not use books, notes, calculators, computers, tablets, or phones.
- Write legibly. Cross out any work that you do not wish to have scored.
- Show all of your work and thoroughly explain your reasoning. Unsupported answers may not earn credit.

Scoring

Question	Points	Score
1	20	
2	10	
3	10	
4	10	
5	7	
6	13	
7	20	
8	10	
Total:	100	

1. Determine, with justification, whether the given series converges or diverges.

(a) (10 points) $\sum_{n=1}^{\infty} \frac{2 + \sin(e^n)}{n^3 + 1}$

(b) (10 points) $\sum_{n=5}^{\infty} \frac{1}{n \ln n}$

2. (10 points) Determine the value of the following infinite sum. You do not have to justify the fact that this converges, and your answer can be left in unsimplified form.

$$\sum_{n=4}^{\infty} \left(\frac{3^{n-1}}{5^{n+2}} - \frac{2 \cdot 4^n}{3 \cdot 7^{n-2}} \right)$$

3. (10 points) Find the interval of convergence of the following power series.

$$\sum_{n=0}^{\infty} \frac{3^{n-1}}{n^2 + 1} (x - 3)^n.$$

4. (10 points) Find a power series representation of the function $f(x) = \frac{x}{(1-2x)^3}$ and determine its radius of convergence. (Just the radius, not the full interval of convergence.)

Hint: Start by differentiating $\frac{1}{1-y}$ twice with respect to y .

5. (7 points) Find the Taylor series of the function $f(x) = \frac{1}{x}$ centered at 5.

6. (13 points) Justify the following equality

$$\int_0^1 \sin(x^3) dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!(6n+4)}$$

and determine the partial sum of the series on the right with the fewest number of terms which approximates the integral on the left to within an error of $\frac{1}{100,000}$.

Hint: Start by finding a power series representation of $\sin(x^3)$. For the second part of the problem, take it for granted that $\frac{1}{7! \cdot 22} < \frac{1}{100,000} < \frac{1}{5! \cdot 16}$.

7. Find the general solution of each of the following nonhomogeneous differential equations.

(a) (10 points) $y'' + 10y' + 25y = 5x - e^{2x}$

(b) (10 points) $y'' + 3y = 7 \sin x + \cos x$

8. (10 points) Find the 5th order Taylor polynomial centered at 0 of the solution of the following differential equation with given initial conditions:

$$y'' + x^2 y' - 4y = 0, \quad y(0) = 2, \quad y'(0) = 5$$

Initials:

YOU MUST SUBMIT THIS PAGE.

If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score.

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