Northwestern University

Math 226 Midterm 1
Fall Quarter 2019
October 22, 2019

Name:

Initials: netID:

Instructions

- This examination consists of 7 pages, not including this cover page. Verify that your copy of this examination contains all 7 pages. If your examination is missing any pages, then obtain a new copy of the examination immediately.
- Enter your initials in the indicated box on each page, and enter your Name and netID on the indicated boxes on the cover sheet.
- This examination consists of 5 questions for a total of 100 points.
- You have 50 minutes to complete this examination.
- Do not use books, notes, calculators, computers, tablets, or phones.
- Write legibly. Cross out any work that you do not wish to have scored.
- Show all of your work and thoroughly explain your reasoning. Unsupported answers may not earn credit.

Scoring

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. Provide justification as to why each of the following sequences converge.

   (a) (10 points) $a_n = \sin[(1+n)^{1/n}]$

   (b) (10 points) $b_n = \frac{5\sin^2(n^2)+3\cos(n^3)}{n+1}$

2. (20 points) Justify the fact that the following series converges, and determine the value to which it converges. Your answer for the value should be left in unsimplified form.

$$\sum_{n=3}^{\infty} \frac{2 \cdot 3^{n+2}}{5 \cdot 7^{n-1}}$$

3. (20 points) Use the integral test to verify that the following series converges. Be sure to explain why the integral test is applicable. (Note: integration by parts is NOT needed.)

$$\sum_{n=4}^{\infty} ne^{-n^2}$$

4. Determine whether each of the following series converge or diverge. Justify your answer and be clear about which convergence test you are using.

   (a) (10 points) $\sum_{n=1}^{\infty} a_n$, where a_n is the sequence from Problem 1 part (a).

   (b) (10 points) $\sum_{n=2}^{\infty} \frac{n^3 + 2n}{n^9 + 1}$

5. (20 points) This problem deals with the following alternating series:

$$\sum_{n=0}^{\infty} \frac{(-1)^n(n+2)}{e^{2n}}$$

Verify that this series converges, and find a restriction on how large the difference between the 3-rd partial sum of this series and the actual value to which it converges can be. (This restriction can be left in an unsimplified form.)

YOU MUST SUBMIT THIS PAGE.

If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score.

YOU MUST SUBMIT THIS PAGE.

If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score.