

# Northwestern University

Math 226 Midterm 2

Fall Quarter 2019

November 18, 2019

Name:

Initials:

netID:

## Instructions

- This examination consists of 7 pages, not including this cover page. Verify that your copy of this examination contains all 7 pages. If your examination is missing any pages, then obtain a new copy of the examination immediately.
- Enter your initials in the indicated box on each page, and enter your Name and netID on the indicated boxes on the cover sheet.
- This examination consists of 5 questions for a total of 100 points.
- You have 50 minutes to complete this examination.
- Do not use books, notes, calculators, computers, tablets, or phones.
- Write legibly. Cross out any work that you do not wish to have scored.
- Show all of your work and thoroughly explain your reasoning. Unsupported answers may not earn credit.

## Scoring

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. (20 points) Find the interval of convergence of the following power series.

$$\sum_{n=0}^{\infty} \frac{3^n}{2^{n-1}(n+5)} (x-1)^n$$

2. Find power series which represent each of the functions below around the given centers.

(a) (10 points)  $f(x) = x^2 \sin(x^5)$  centered at 0

(b) (10 points)  $g(x) = \int_0^x \frac{1}{2-(t-3)^4} dt$  centered at 3 (Hint: factor 2 out of the denominator of the function being integrated.)

3. (a) (10 points) Find the third-order Taylor polynomial centered at 2 of the function  $f(x) = \sqrt{1+x}$ .

- (b) (10 points) Find the Taylor series centered at 1 of the function  $g(x) = x + x^2 - 2x^3$ .

4. Consider the function  $f(x) = e^{3x}$

- (a) (10 points) Determine the maximal error which arises when approximating  $f(x)$  using the polynomial  $1 + 3x + \frac{9}{2}x^2$  for  $x$  in the interval  $(-2, 2)$ .

- (b) (10 points) Find the values of  $x$  for which the error in approximating  $f(x)$  using the polynomial  $1 + 3x$  is at most  $\frac{1}{100}$ . Your answer should be an interval characterizing such  $x$ , but the endpoints of this interval do not have to be written in simplified form.

5. Find all solutions of the following second-order differential equations.

(a) (10 points)  $y'' - 3y' - 28y = 0$

(b) (10 points)  $y'' - 4y' + 5y = 0$

Initials:

**YOU MUST SUBMIT THIS PAGE.**

If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score.

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