Northwestern

Math 290-2 Midterm 1
Winter Quarter 2019
February 4, 2019

Last name: ______________________ Email address: ______________________

First name: ______________________ NetID: ______________________

Instructions

- Mark your instructor's name.

  _____ Cañez

  _____ Newstead

  _____ Norton

- This examination consists of 11 pages, not including this cover page. Verify that your copy of this examination contains all 11 pages. If your examination is missing any pages, then obtain a new copy of the examination immediately.
- This examination consists of 6 questions for a total of 100 points.
- You have one hour to complete this examination.
- Do not use books, notes, calculators, computers, tablets, or phones.
- Write legibly and only inside of the boxed region on each page.
- Cross out any work that you do not wish to have scored.
- Show all of your work. Unsupported answers may not earn credit.

1. Determine whether each of the following statements is **TRUE** or **FALSE**. Justify your answer. (This question has **four** parts. Remember: A statement is "true" if it is always true. If not, it is "false.")

   (a) (5 points) There exist nonzero vectors $\vec{a}$ and $\vec{b}$ in $\mathbb{R}^3$ such that $\vec{a} \times \vec{b} = \vec{a}$.

   Answer:

   (b) (5 points) Let A be an $n \times n$ matrix with columns $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$. If $\vec{v}_i \cdot \vec{v}_j = 0$ for all $i \neq j$, then $A^T A$ is diagonal.

   Answer:

(c) (5 points) Orthogonal projection onto a plane is an orthogonal transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$.

Answer:

(d) (5 points) There exists a 2×2 orthogonal matrix Q such that $Q^T \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} Q = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$.

Answer:

2. Determine whether each of the following statements is **ALWAYS** true, **SOMETIMES** true, or **NEVER** true. Justify your answer. (This question has **four** parts.)

   (a) (5 points) Let A be a 3×3 matrix. For all $\vec{x}$ in $\mathbb{R}^3$, $\vec{x} \cdot A\vec{x} = 3x^{10} + 4xy - 6yz + 2z^3$.

   Answer:

   (b) (5 points) Let A be a 2×2 orthogonal matrix, and let $S = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Then $S^{-1}AS$ is an orthogonal matrix.

   Answer:

(c) (5 points) Suppose A is an $m \times n$ matrix and $\vec{b}$ is a vector in $\mathbb{R}^m$. If $\vec{b}$ is perpendicular to every column of A, then the zero vector in $\mathbb{R}^n$ is a least squares solution to $A\vec{x} = \vec{b}$.

Answer:

(d) (5 points) Let k be a real number and let A be a 3×3 matrix with eigenvectors

$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ k \end{bmatrix}$$

corresponding to eigenvalues $1, 1, 2$, respectively. Then A is symmetric.

Answer:

3. (15 points) Find the point(s) on the curve $6x^2 + 4xy + 3y^2 = 1$ which are closest to $(0, 0)$.

4. (12 points) Find the function of the form $f(t) = c_0 + c_1 t$ that best fits the data points $(0, -3), (1, -3)$, and $(1, -6)$.

5. (15 points) Consider the intersecting lines $l_1 : x = 1+t, y = 6+5t, z = -2t$ and $l_2 : x = -3+3t, y = t, z = 1+t$. Find parametric equations for the line that intersects l_1 and l_2 at their point of intersection and is perpendicular to both l_1 and l_2.

6. (18 points) Find an orthonormal basis for $V = (\text{im } A)^{\perp}$, where A is the 4×2 matrix defined by

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 3 \\ -1 & -1 \end{bmatrix}.$$

(Recall: If W is a subspace of $\mathbb{R}^n$, then $W^{\perp} = \{\vec{x} \in \mathbb{R}^n \mid \vec{x} \cdot \vec{w} = 0 \text{ for all } \vec{w} \text{ in } W\}$.)

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